THE JOURNAL
OF THE
ACOUSTICAL SOCIETY OF AMERICA

Volume 10 Number 2

OCTOBER • 1938

The Absorption of Sound in Carbon Dioxide and Other Gases

VERN O. KNUDSEN AND EDWIN F. FRICKE
University of California, Los Angeles, California

(Received July 6, 1938)

Sound absorption measurements in pure CO_2 at atmospheric pressure and 22°C have been conducted which, together with measurements by others, confirm the collision theory of anomalous absorption as developed by Einstein, Kneser, and others. The absorption coefficient is appreciable at frequencies as low as 2000 cycles, and increases to a maximum of 0.317 per wave-length at 77,000 cycles. This maximum is higher than values obtained by previous workers, and indicates that both the deformation and symmetrical valence vibrations participate in the exchanges between translational and vibrational energy. Small impurities, as water or alcohol vapors, affect the absorption greatly; the entire absorption band is shifted to higher frequencies. Measurements in mixtures of CO_2 in O_2 and CO_2 in N_2 indicate that neither the O_2 nor N_2 is appreciably excited by collisions with CO_2. Measurements in CS_2 reveal an absorption similar to that for CO_2 except that the absorption begins at about 10,000 cycles. In mixtures of CS_2 and O_2, the observed absorption at frequencies below 10,000 cycles is accounted for by assuming that only the vibration of O_2 molecules is excited by collisions with CS_2 molecules; at higher frequencies the CS_2 molecules also are excited, principally by collisions with other CS_2 molecules.

THE experiments reported in this paper are a continuation of a general investigation of the absorption of sound in gases.[1] The apparatus and technique used are essentially the same as those described by Knudsen and Obert.[1] Briefly, the absorption of a gas is determined by measuring the average intensity of diffuse sound in a two-foot cubical chamber, first when the chamber is filled with a gas of small but known absorption, and then when it is filled with a gas the absorption of which is to be determined.

In the present experiments it was necessary to oscillate the crystal microphone, and to take into account the rate of emission of the source of sound as influenced by changes in (1) the density of the gas and (2) the velocity of sound in the gas. This was not necessary in the earlier experiments since the composition of the gas was altered only by the presence of small impurities, and hence the density and sound velocity remained practically constant.

The arrangement of the apparatus is indicated in Fig. 1. The sound source was a Pierce Magnetostriction oscillator for frequencies above 7000 cycles and a Western Electric 555 electrodynamic loudspeaker for lower frequencies. The microphone, mounted on a hollow brass sphere 10 cm in diameter, was swung on one end of a 28-cm tube, through an arc of 270 degrees, and with such a speed that it made a complete cycle in two seconds. The microphone, of the Brush

[1] Knudsen and Obert, J. Acous. Soc. Am. **7**, 249–253 (1936), and earlier papers by Kneser and by Knudsen in this journal.

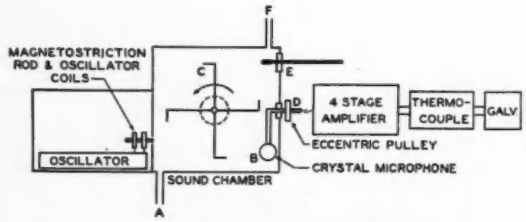


FIG. 1. Schematic arrangement of apparatus.

crystal type, was insulated from the spherical mounting by means of cotton packing. It was propelled, as were also the diffusing paddles within the chamber, by a motor and eccentric devised by Mr. Russell Neil. The movement of the microphone was sufficient to yield intensity measurements which represented satisfactorily the average intensity throughout the chamber, provided the absorption in the gas was not so great as to prevent multiple reflections from the boundaries of the chamber, that is, provided the sound was diffuse.

In the experiments on carbon dioxide, the percentage of the gas added to the chamber was measured by a calibrated flow meter, and constantly checked by weighing specimens of the gas mixture, in which case the lag of density between the gas in the weighing bulb and that in the chamber was appropriately corrected. The percentage of other gases added to the chamber was determined by weighing or measuring the amount of added gas or vapor.

The gases in the chamber were dried by passing them through concentrated sulphuric acid and then through dry phosphorous pentoxide. Unless this is done for commercial CO_2, the water impurity gives rise to troublesome irregularities in the absorption. Care was taken to replace all the gas inside the drying apparatus with carbon dioxide (or other gas under test) before the flow meter was connected to the chamber. Before any absorption measurements were made, the flow of basic gas (N_2 or O_2) was continued long enough to remove all undesirable traces of impurities, such as H_2O, CO_2, CS_2, etc.

The extraneous noise level within the chamber, owing largely to the rotation of the diffusing paddles, and the intensity of the source of sound was checked at the beginning and end of each run; in general, both remained very constant, and by the use of band-pass filters in the detector circuit the noise level was inappreciable at all frequencies except 2000 cycles. The batteries, temperature, and barometer readings were checked frequently during the progress of the runs. It was necessary to eliminate all stray induction and pick-up, such as that from the vibrating source and from the electric motor driving the fan and microphone.

The reliability of the apparatus was tested by trial measurements in oxygen containing small amounts of water vapor. The absorption coefficients determined by these preliminary tests were in good agreement with those which had been determined previously by the decay method.[2] Further, the accuracy with which separate sets of measurements on the same gas mixture were duplicated, even after the elapse of more than a month in some cases, gave assurance that the apparatus and method of measurement were reliable. For example, three separate sets of measurements were made for carbon dioxide in oxygen at 7965 cycles, and the mean deviation of separate observations of sound pressure for the same gas composition was less than 2.0 percent.

EQUATIONS FOR CALCULATION OF ABSORPTION COEFFICIENTS

The rate of emission of sound energy from the high frequency source—a magnetostriction rod which is mounted so that it acts as a piston in a plane wall—will depend upon the velocity of the "piston" U (which is maintained constant), the radiation resistance, the wave-length λ, and the radius of the piston R. According to Crandall,[3] the rate of emission W for such a piston source is

$$W = U^2 \cdot \pi R^2 \cdot \rho c [1 - J_1(2kR)/kR], \qquad (1)$$

where J_1 is a Bessel function, ρ is the density of the gas in the chamber, c the velocity of sound in the gas, and $k = 2\pi/\lambda$. For the Western Electric 555 electrodynamic loudspeaker as the source of sound, the power radiated was calculated by means of the formula

$$W = \rho c U_0^2 / S_0 (1 - n^2/4k^2)^{\frac{1}{2}}, \qquad (1')$$

[2] Knudsen, J. Acous. Soc. Am. **5**, 112–121 (1933).
[3] Crandall, *Theory of Vibrating Systems and Sound*, (Van Nostrand, 1926), p. 146.

where S_0 = area of throat, and n is the exponential constant of the horn.[4]

The sound intensity in the chamber is determined by means of the Brush crystal microphone, approximately 0.6 cm square, and its associated amplifier, thermocouple, and galvanometer. This apparatus, as calibrated, measures the sound pressure acting on the crystal. The sound pressure in the chamber will be somewhat less than that on the crystal, owing to diffraction and reflection of sound from the microphone and its spherical mounting; but for wave-lengths which are appreciably shorter than the dimensions of the spherical microphone, that is, for frequencies above about 10,000 cycles, the pressure measured by the microphone will be approximately double that in the chamber. For frequencies below 10,000 cycles, the microphone response will gradually diminish, until at very low frequencies the pressure registered by the microphone will be the same as that within the chamber.[5] For frequencies below 10,000 cycles it therefore is necessary to apply a correction to the microphone readings, provided the wave-length changes appreciably in a series of measurements. Where such corrections have been necessary, they have been made in accordance with Morse's Eq. (29.12) for a small microphone mounted on a sphere of 5.08 cm radius.[5]

For plane waves, the intensity I will be related to the sound pressure P (in the gas) by the well-known relation $I=P^2/\rho c$. This same relation holds approximately for the average intensity of diffuse sound in a chamber, and will be assumed in the present case.

The average intensity of sound in the chamber is equal to the rate of emission of sound energy in the chamber divided by the total absorption. Thus,

$$I=W/(\alpha S+4mV); \qquad (2)$$

α = the absorption coefficient of the interior boundaries of the chamber ($\alpha=0.018$ for 4000 cycles, 0.019 for 8000 cycles, and 0.020 for 16,000 cycles)

S = interior surface of the chamber $=2.23\times10^4$ cm^2

m = absorption coefficient per cm for the gas

V = volume of chamber $=2.27\times10^5$ cm^3.

When the chamber is filled with gas A (for the rod source)[6]

$$I_a=\frac{P_a^2}{\rho_a c_a}=\frac{U^2\pi R^2\rho_a c_a[1-J_1(2k_aR)/k_aR]}{\alpha S+4m_aV}. \qquad (3)$$

(The subscripts a refer to gas A.) When filled with another gas, or gas mixture, B

$$I_b=\frac{P_b^2}{\rho_b c_b}=\frac{U^2\pi R^2\rho_b c_b[1-J_1(2k_bR)/k_bR]}{\alpha S+4m_bV}. \qquad (4)$$

Dividing (3) by (4), and solving for m_b,

$$m_b=\frac{1}{4V}\left\{\frac{P_a^2}{P_b^2}\left[\frac{\rho_b c_b}{\rho_a c_a}\right]^2\frac{[1-J_1(2k_bR)/k_bR]}{[1-J_1(2k_aR)/k_aR]}\times(\alpha S+4m_aV)-\alpha S\right\}. \qquad (5)$$

If gas A has a small absorption coefficient, the term $4m_aV$ (the absorption in the gas) will be small compared with αS (the absorption by the walls of the chamber). For such gases as nitrogen or oxygen it is so small at the frequencies used in these experiments that the uncertainty in the values of m_a will not appreciably affect the accuracy of the values of m_b computed from (5). However, the preliminary experiments in oxygen containing small amounts of water vapor provided data from which it was possible to calculate, by means of (5), the absorption coefficient for dry oxygen at frequencies of 7965 and 15,800 cycles, assuming that the values of $m_{\max}$ at these frequencies for oxygen containing water vapor, as determined previously by the decay method, are correct. This assumption seems justifiable, since we have repeatedly obtained values for $m_{\max}$ which agree with each other, and also with the values computed for oxygen from the molecular collision theory. The values of m for dry oxygen, determined in this way, are 2.92×10^{-5} cm^{-1} at 7965 cycles and 11.1×10^{-5} cm^{-1} at 15,800 cycles. These values are approximately 50 percent greater than those calculated from classical theory, and conform very well with the classical requirement that the absorp-

[4] See Crandall, reference 3, p. 161, and West, *Acoustical Engineering* (Pitman), p. 150.

[5] See Ballantine, J. Acous. Soc. Am. **3**, 345 (1932); also Morse, *Vibrating Systems and Sound* (McGraw-Hill, 1936), p. 270.

[6] These equations must be modified, in accordance with Eq. (1′), when the electrodynamic source is used.

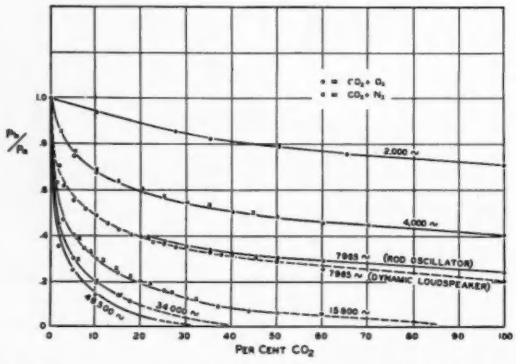


FIG. 2. Curves showing the relative pressures (as measured by the microphone) for CO_2 in O_2 and CO_2 in N_2.

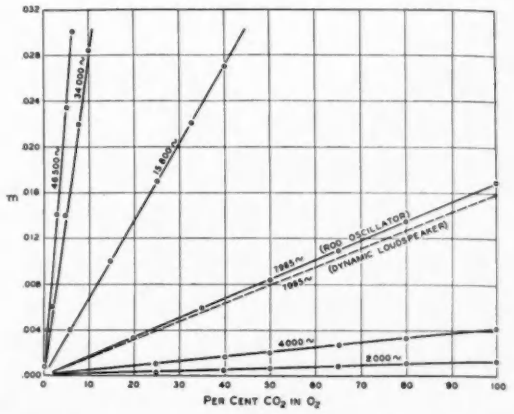


FIG. 3. Graphs showing how the absorption coefficient m depends upon the concentration of CO_2 in O_2 for frequencies between 2000 and 46,500 cycles.

tion coefficient varies directly with the square of the frequency. These values of m for dry oxygen are used for m_a in (5). At other frequencies, the values of m_a for dry oxygen are derived from the measured values at 7965 and 15,800 cycles, assuming that m_a varies directly with the square of the frequency.

The principal experimental findings are as follows:

(a) Absorption of CO_2 in O_2 and CO_2 in N_2

The chamber is first filled with dry O_2 (commercial, 99.6 percent pure) or N_2 (commercial, 99.7 percent pure). Then dry CO_2 (commercial, 99.8 percent pure) is added to the chamber, the gas mixture being maintained at external atmospheric pressure. (In all the following experiments the gas was maintained at room temperature, 22 to 23 degrees C.) The observed[7] relative sound pressures P_b/P_a in the gas, for pure tones of 2000, 4000, 7965, 15,800, 34,000, and 46,500 cycles, are shown in Fig. 2. At 7965 cycles, measurements were made with the rod oscillator and also with the dynamic speaker as the source. It will be noted that the absorption is essentially the same whether CO_2 be added to O_2 or to N_2. This was rather surprising, since at these frequencies O_2 containing small impurities of CO_2 was expected to exhibit the usual absorption of O_2 containing such impurities as H_2O, NH_3, and other gases investigated by Kneser and Knudsen.[8] Apparently, collisions between CO_2 and O_2 are not effective in exciting the vibration of O_2 molecules. The absorption appears to depend entirely on the CO_2. The measurements at lower frequencies indicate that the anomalous absorption in CO_2 begins as low as 2000 cycles.

The graphs for the absorption coefficient of CO_2 in O_2, based on the data in Fig. 2, are shown in Fig. 3. (These same graphs would apply to CO_2 in N_2.) For frequencies of 2000, 4000, and 7965 cycles, the graphs in Fig. 3 give the absorption coefficient for all mixtures of CO_2 and O_2 from 100 percent O_2 to 100 percent CO_2. Since these graphs are straight lines throughout their entire course, it seems justifiable to extrapolate the graphs for 15,800, 34,000, and 46,500 cycles to 100 percent CO_2. (The graphs are straight lines throughout those portions for which we could make measurements.) At the frequencies of 34,000 and 46,500 cycles it was assumed that the sound in the chamber was diffuse only for concentrations of CO_2 less than ten percent.

By making these extrapolations it is possible to obtain experimental values for m_b and $\mu(=m\lambda)$ as given in Table I.

TABLE I.

FREQUENCY, ν	m_b	μ
2,000	0.0017	0.023
4,000	.0043	.028
7,965	.017	.058
15,800	.068	.117
34,000	.285	.229
46,500	.455	.270

[7] For all frequencies the *observed* values of P_b/P_a must be corrected (in accordance with the theory referred to in note 5) before they are used in Eq. (5).

[8] Kneser and Knudsen, Ann. d. Physik [5] **21**, 682–696 (1935).

These values of μ are plotted in Fig. 4, in which also are included data at higher frequencies (above 89 kc) obtained by Curtis.[9] Data on the absorption in CO_2 obtained by other investigators are not included, since it is probable that such data are not for entirely dry CO_2; the presence of even very small impurities of H_2O will shift the entire absorption band to higher frequencies. With the exception of the one Curtis value at 89 kc, his data and those we have obtained fit reasonably well an absorption curve of the Kneser type, provided the maximal value of μ is 0.317 and the frequency at which the absorption is a maximum is 77 kc.

These values of μ are higher than those considered by Kneser[10] (based on the assumption that only the deformation vibration of CO_2 was involved) when he attempted to adapt a theoretical curve to the data given by Abello, Pielemeier, and Grossman. Kneser assumed that the absorption was a maximum at 250 kc, but this was based on data for CO_2 which was not free from H_2O. More recent investigations, especially those by Grossmann and Curtis, have indicated that the absorption maximum occurs at approximately 100 kc, and our data indicate that it occurs at a somewhat lower frequency, namely 77 kc; although it should be borne in mind that both μ_{max} and ν_{max} in Fig. 4 were obtained by extrapolating our data and Curtis' data to a point of inflection. Our present apparatus is not suitable for measurements at frequencies above 46,500 cycles, but apparatus is being designed in this laboratory with which such measurements can be made.[11] It then will be possible to determine more accurately the shape of the absorption curve for CO_2 for frequencies between 50 and 200,000 kc, that is, in the vicinity of maximal absorption.

However, it is of interest to compare μ_{max} in the absorption curve of Fig. 4 with possible values which are predicted by the collision theory. If we assume only two body collisions between the CO_2 molecules, Bourgin's absorption equation (0.5)[12] becomes identical with the better known Kneser equation, namely[13]

$$\mu = m\lambda = 2\pi \frac{RC_i}{C_\infty(R+C_\infty)} \frac{\omega k_{10}}{\omega^2 + k_{10}^2} \qquad (6)$$

in which we have assigned the following values:

R = the gas constant = 1.98 calories/mole
C_i = vibrational heat capacity (which may include the valence as well as the deformation vibrations. This will be discussed later.)
$C_\infty = C_p - (R + C_i)$ = the molecular heat at constant volume at infinite sound frequency.
$C_p = 7.0 + 0.0071T - 0.00000186T^2$.[14]
T = absolute temperature.
$\omega = 2\pi$ times the sound frequency.
k_{10} = the reaction rate constant, that is, the number of transitions per molecule per second from the excited to the normal state.

The absorption is a maximum when $\omega = k_{10}$, in which case

$$\mu_{max} = RC_i / C_\infty(R + C_\infty). \qquad (7)$$

If we assume that the collisions between CO_2 molecules excite only the deformation vibration, C_i is computed from the characteristic temperature E/R (E = vibrational energy per mole = 1930 for the deformation vibration, and hence E/R

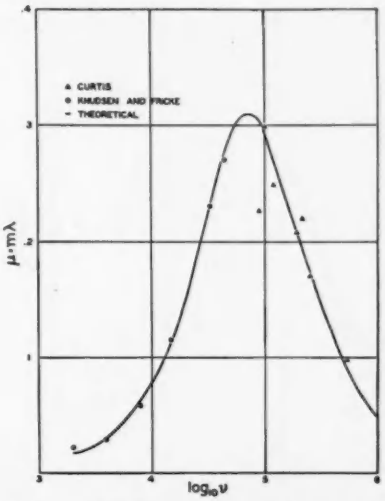


FIG. 4. Curve showing the absorption per wave-length for CO_2 assuming that the maximal absorption occurs at 77 kc, and that the maximal absorption is 0.317.

[9] Roger W. Curtis, Phys. Rev. **46**, 811–815 (1934).
[10] Kneser, Ann. der Physik (5) 16, 337–349 (1933).
[11] Just as this article was ready for publication, N. Schmidtmüller, Akust. Zeitschrift **3**, 115–129 (1938) reports measurements on CO_2 for frequencies between 20 and 100 kc, but his dryest CO_2 contained 2.1 g/m³ of water vapor and hence no measurements were made on *dry* CO_2. By a process of extrapolation, Schmidtmüller indicates that dry CO_2 would have its maximal absorption at 100 kc.
[12] D. G. Bourgin, Phys. Rev. **50**, 355–369 (1936).
[13] See Eq. (3) Ann. d. Physik (5) **21**, 684 (1935).
[14] Lewis and Randall, *Thermodynamics* (1923), p. 80.

$=974°A$).[15] Thus, for $T=296°A$, the contribution to C_i of the deformation vibration C_d, is

$$C_d=2R(974/296)^2e^{-974/296}$$
$$=1.598 \text{ cal./gram-mole.}$$

For the symmetrical valence vibration, E/R is 1838°A, or the corresponding heat capacity C_s is 0.308 cal./gram mole; and for the asymmetrical valence vibration E/R is 3367°A, or the corresponding heat capacity is $C_a \sim 0.0068$ cal./gram mole. Now, if C_d be used for C_i in Eq. (7) the value of μ_{max} becomes 0.241 which certainly does not accord with our data. But if we use C_d+C_s for C_i, that is, if the symmetrical valence as well as the deformation vibration be assumed to share in the energy exchanges (the asymmetrical vibration term is too small to justify any conclusion respecting its participation, but it is generally assumed that this vibration is not excited at room temperatures), the value of μ_{max} from Eq. (7) becomes 0.317. It will be seen from Fig. 4 (in which the curve was computed from Eq. (6)) that this value at a frequency of 77 kc accounts very well for our experimental measurements, and also reasonably well with the measurements of Curtis. It would seem justifiable therefore to conclude that both the valence and the deformation vibrations participate in the energy exchanges which accompany collisions between CO_2 molecules. This is consistent with the dispersion and absorption measurements of Eucken and Nümann[16] which indicate that for CO_2 the valence vibrations are excited as easily as are the deformation vibrations. Further, there is no evidence that there are different reaction constants which characterize separately the deformation and the valence vibrations.

The presence of the normal amount of CO_2 in the air is sufficient to produce a marked increase in the absorption above that which is characteristic of O_2 and N_2, free from CO_2. Thus, at 46,500 cycles, the absorption measurements in air compared with those for pure, dry O_2 or N_2 indicated that the air contained 0.035 percent CO_2, which is consistent with the concentration of CO_2 often found in air. In fact, sound absorption measurements at appropriate frequencies furnish a delicate test for the quantitative determination of small impurities of CO_2 in such gases as N_2, O_2, or air. Thus, at 46,500 cycles, and for small impurities of CO_2 in air, the deflection of the galvanometer changed as much as 1.0 cm for a change in concentration of CO_2 of one part in 20,000. It is probable that even smaller changes in concentration could be detected by this method at the frequency of maximal absorption, that is, about 77 kc.

(b) Effect of impurities on the absorption of sound in CO_2

It has been shown repeatedly that the effect of certain impurities in CO_2, O_2, and other gases is to shift the absorption or dispersion band to higher frequencies. It is to be expected therefore that the addition to CO_2 of such impurities as water vapor, alcohol, or hydrogen should reduce the absorption for the frequencies used in the present experiments, that is, frequencies which are on the ascending branch of the absorption curve shown in Fig. 4. On the other hand, for frequencies which are on the descending branch of the absorption curve, the addition of these same impurities should increase the absorption; this was confirmed by the earlier experiments with O_2 [1, 8] in which case it was possible to investigate the effect of impurities throughout the entire frequency band of the anomalous absorption.

The effect of adding water vapor to CO_2 at frequencies of 7965 cycles and 15,800 cycles is shown in Figs. 5 and 6, respectively. The intensity of the sound increases, that is, the absorption decreases, as more and more water vapor is added to the chamber. Only 0.55 percent of water vapor is sufficient at 7965 cycles to make the CO_2 as "transparent" as dry O_2; additional water vapor further reduces the absorption but there is a limit beyond which the absorption does not diminish appreciably. Apparently, the absorption band has been shifted so far toward higher frequencies that at 7965 cycles the absorption in CO_2 containing one percent or more of water vapor consists almost exclusively of the classical type. This does not appear to be the case of 15,800 cycles, since at this frequency the "transparency" approaches

[15] R. Mecke, Zeits. f. physik. Chemie **B16**, 421 (1932).
[16] A. Eucken and E. Nümann, Zeits. f. physik. Chemie **B36**, 163–183 (1937).

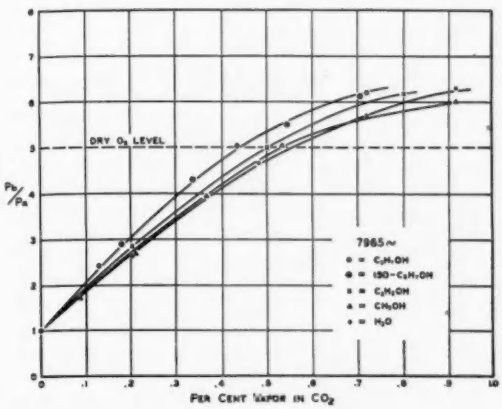


FIG. 5. Curves showing the relative sound pressure for a frequency of 7965 cycles for CO_2 containing the indicated impurities.

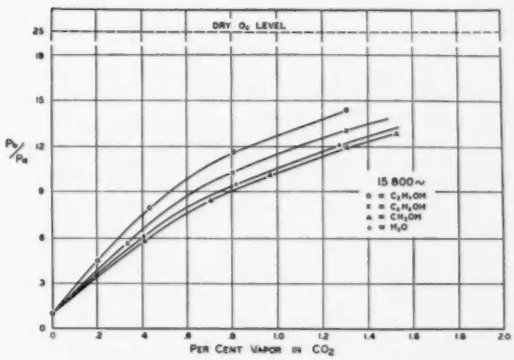


FIG. 6. Curves showing the relative sound pressure for a frequency of 15,800 cycles for CO_2 containing the indicated impurities.

but does not reach that of dry O_2. These findings are in approximate agreement with those of Rogers.[17]

It is possible to displace horizontally the absorption curve shown in Fig. 4 so that it will pass through the absorption values calculated from the CO_2 plus H_2O curves in Figs. 5 and 6, from which we can calculate certain transition constants for CO_2 containing small amounts of water vapor, as has been done by Eucken and his associates from dispersion measurements;[17] this will not be attempted until further data have been obtained at higher frequencies. The present findings, however, point to conclusions which agree at least qualitatively with those reached by Eucken and his associates.

The anomalous absorption in CO_2, and the influence of impurities on this absorption, can be demonstrated effectively by means of a piece of water pipe (about 10 feet long and 2 inches in diameter) and a high frequency but audible source of sound (9000 to 10,000 cycles). If such a source of sound be attached to one end of the pipe when it is filled with dry air (or N_2 or O_2) the sound will be transmitted through the pipe with very little attenuation, so that it will be plainly audible to listeners near the other end of the pipe. If the pipe be filled with dry CO_2 there will be a marked "quenching" of the tone, but if now a small amount of water vapor be added to the CO_2 the tone will be heard even louder than when the pipe was filled with dry air.

In Fig. 5 are shown also the effects of adding different types of alcohol vapor to CO_2 at 7965 cycles. (The data in Fig. 5 enable us to calculate the coefficient m for CO_2 containing the impurities indicated. These values of m are presented in Fig. 7.) The effect of the alcohol impurities is similar to that obtained for water vapor. It will be noticed, however, that the heavier alcohols are more effective in altering the absorption in CO_2 than are the lighter ones. Thus, as is shown best in Fig. 5, at 7965 cycles the addition of 0.55 percent of water vapor, 0.53 percent of methyl alcohol, 0.50 percent of ethyl, or 0.43 percent of either iso- or *n*-propyl alcohol is just sufficient to restore the sound intensity in the chamber to that which results when the chamber is filled with dry O_2. Repeated trials with iso- and *n*-propyl alcohols, which differ only in the location of the OH group on the carbon chain, indicate that the location of this group has no apparent effect on the absorption.

The effects of HCl and H_2 as impurities in CO_2 have also been investigated. HCl is less active than H_2O but more active than H_2. The results confirm the findings of Eucken and Becker[17] respecting the effects of these impurities on the dispersion curve for CO_2.

The absorption effects of adding *n*-propyl or ethyl alcohol as impurities to dry O_2 at 7965 cycles are shown in Fig. 8. As in the case when

[17] Phys. Rev. **45**, 208–211 (1934). Rogers' findings indicate that CO_2 containing about 1.0 percent of H_2) has its maximal absorption at 410 kc; that is, the 1.0 percent impurity of H_2O has shifted the absorption band somewhat more than two octaves, from 77 to 410 kc. This shift also agrees approximately with that in the dispersion for CO_2 plus H_2O, observed by Eucken and Becker, Zeits. f. physik. Chemie **B27**, 235 (1934); also Eucken and Nümann, Zeits. f. physik. Chemie **B36**, 163 (1937).

CO_2 was the principal gas, the heavier alcohol is more effective in disturbing the absorption than is the lighter one. Thus, a concentration of only 0.15 percent of the *n*-propyl was sufficient to shift the absorption to its maximal value (minimal intensity), but 0.225 percent of the ethyl was required to produce the same shift. It will be noticed that each curve has its minimum at the same sound intensity; this is in agreement with the earlier findings of Kneser and Knudsen.[8] Also, the value of the maximal absorption coefficient for O_2, computed from these curves by means of Eq. (5), agrees well with that obtained by the reverberation method.[2]

It is of interest to compare the effects of the same impurities in O_2 and in CO_2. Thus, methyl alcohol as an impurity in CO_2 exhibits about the same degree of activity in disturbing the absorption as does water vapor, that is, about equal amounts of either methyl alcohol or water vapor produce the same effect on the absorption; but this same alcohol as an impurity in O_2 is about three times more active than is water vapor in O_2.[8] These results are consistent with the chemical affinities of the colliding molecules; since methyl alcohol and carbon dioxide mixtures have no possibility of reaction, the absorption is primarily the result of collisions between the two kinds of molecules. In mixtures of water and CO_2 there might be formed small quantities of carbonic acid, as a result of the small negative free energy (about 43 cal.) for the reaction, but there is very little possibility that any reaction at all will occur in the short time (less than one hour) required to make a series of measurements. On the basis of chemical affinity, therefore, we might expect alcohol and water to produce approximately the same effects on the vibration of CO_2 molecules.

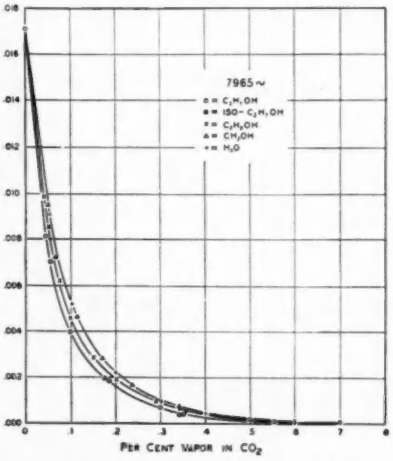


FIG. 7. Curves showing the absorption coefficient at 7965 cycles of CO_2 containing the indicated impurities.

In the case of gas mixtures of water vapor and O_2 there is a very small probability that any hydrogen peroxide molecules will be formed due to the large positive free energy of reaction (59,554 cal.). On the other hand, if we assume that there is a tendency toward reaction between the alcohol and oxygen molecules, we can account

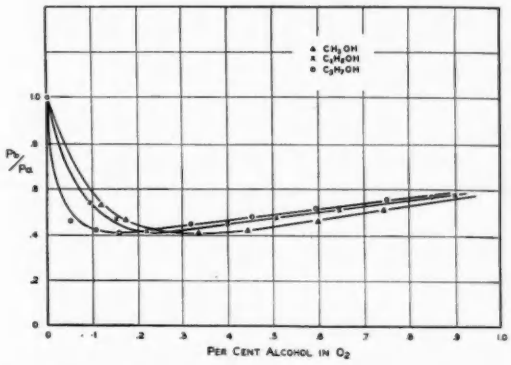


FIG. 8. Curves showing the relative sound pressure in oxygen containing the indicated impurities at a frequency of 7965 cycles.

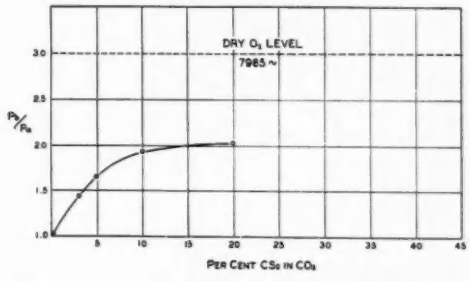


FIG. 9. Curve showing the relative sound pressure for CS_2 in CO_2.

for the observed greater activity of alcohol (compared with water) in disturbing the vibration of oxygen molecules.

The effect of adding CS_2 to CO_2, at 7965 cycles, is shown in Fig. 9. It appears that CS_2 as an impurity is much less effective in disturbing the absorption in CO_2 than is H_2O, H_2, HCl, or any one of the four alcohols reported above, that is, it appears that collisions between CO_2 and CS_2 molecules are not so likely to excite, or

dissipate, the vibration of the CO_2 molecules as are collisions between CO_2 and the other named impurities. The activity of the different impurities investigated line up in the following order: N_2 or O_2 (least active), CS_2, H_2, HCl, H_2O, methyl alcohol, ethyl alcohol, *n*- or iso-propyl alcohol (most active). It will be seen from Fig. 9 that as CS_2 was added to the chamber the sound intensity increased, quite rapidly at first but approached a constant value for concentrations above 15 percent. (Saturation of CS_2 vapor was reached at about 20 percent, at the existing temperature of 22°C.) It is probable, as this curve indicates, that there is some absorption at this frequency which is attributable to CS_2, or to the action between CS_2 and CO_2. In fact, measurements at somewhat higher frequencies (15,800 cycles, for example) give strong evidence of such absorption (see Section (*c*) below). It will be necessary to apply a more general theory of absorption, such as that developed by Bourgin,[12] to account for the absorption in such gas mixtures; but more data in respect to CS_2 are needed before this can be done satisfactorily.

(c) Absorption of carbon disulfide in O_2, N_2, or dry air

In Figs. 10 and 11 are shown the effects of adding carbon disulfide to dry O_2 or dry N_2 at

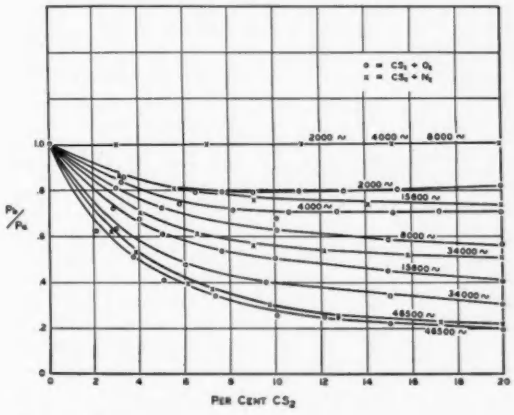


FIG. 10. Curves showing the relative sound pressure for CS_2 in O_2 and CS_2 in N_2 at the indicated frequencies.

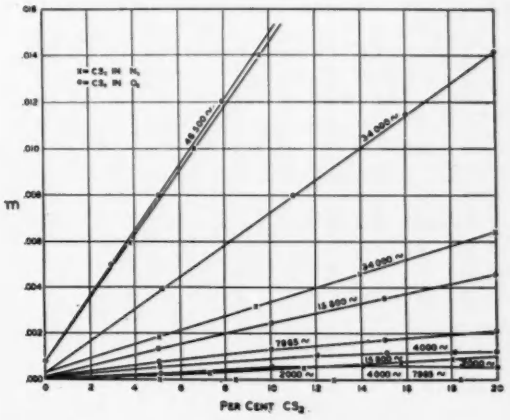


FIG. 11. Curves showing the absorption coefficient m for CS_2 in N_2 and CS_2 in O_2.

frequencies of 2000, 4000, 7965, 15,800, 34,000, and 46,500 cycles. The curves in Fig. 10 show the observed ratios P_b/P_a as a function of the concentration of CS_2 in N_2 or in O_2, and the curves in Fig. 11 give the corresponding values of m. Data for the absorption of CS_2 in dry air, but not included in Figs. 10 and 11, fall between the corresponding data for CS_2 in N_2 and for CS_2 in O_2. The data fail to show any absorption whatever in mixtures of CS_2 and N_2 at frequencies of 2000, 4000, and 7965 cycles, but do show a moderate absorption in these mixtures at 15,800 and 34,000 cycles. On the other hand, the data indicate that the mixtures of CS_2 and O_2 are absorptive at all frequencies. The results can be explained on the assumption that the CS_2 acts as an impurity in O_2, such as water vapor or alcohol, which primarily brings into play the vibrational heat capacity of O_2. This effect is predominant for frequencies below about 15,000 cycles, but for higher frequencies the absorption in the CS_2 becomes predominant. Thus, at 46,500 cycles, the absorption of CS_2 in N_2 is nearly as large as that of CS_2 in O_2. In the case of CS_2 in O_2 it probably will be necessary to invoke a theory of absorption of gas mixtures, such as Bourgin's, to account for the observed effects, especially for frequencies below 46,500 cycles.

Measurement of Absorption in Rooms with Sound Absorbing Ceilings

J. R. POWER
Bell Telephone Laboratories, New York City
(Received July 15, 1938)

Serious difficulties have been encountered in attempts to measure the absorption coefficients of sound absorbing ceilings in large offices. An analysis of the sound field is made and it is concluded (1) that the reverberation time formula is usually invalid if the absorption is concentrated on one surface of the room, (2) that the energy density formula is not subject to such a restriction and (3) that under such conditions there is no apparent field method of determining the absorption coefficient that will serve as an accurate check on laboratory measurements.

INTRODUCTION

OCCASIONS arise when it is desirable to measure the absorption coefficients of sound absorbing materials in commercial installations. The general methods available for this purpose involve the calculation of the absorption from either the reverberation time of the room or from the energy density. The absorption in auditoriums and other more or less cubical rooms can usually be satisfactorily determined by means of commonly accepted reverberation time formulas. When the same method is used to compute the absorption in large offices with low, sound absorbing ceilings very large errors result, the calculated value in some instances being as low as 50 percent of the actual value. This discrepancy does not appear when the energy density equation is used. It therefore seems that the error is due to the reverberation time formula not being adapted to office type rooms with treated ceilings.

The commonly used formulas for the relation of energy density[1] and reverberation time to absorption coefficient are

$$e=4E(1-\alpha)/c\sum(\alpha_n s_n)+E/4\pi D^2 c, \quad (1)$$

$$T=0.05V/-S\log(1-\alpha), \quad (2)$$

where e = energy density of sound in ergs/cc,
E = power of the source in ergs/sec.,
c = velocity of sound in cm/sec.,
s_n = area of a portion of the room surface in cm²,
α_n = absorption coefficient of area s_n,
D = distance from source to point of observation,
V = volume of room in cu. ft.,
T = reverberation time in sec.,
S = total surface of room in sq. ft.,
α = average absorption coefficient.

[1] Olson and Massa—*Applied Acoustics* p. 339.

The reverberation time formula (Eq. 2) is that developed by Eyring[2] and appears superior for our purpose to that used by Sabine,

$$T=0.05V/\alpha S \quad (3)$$

or that proposed by Millington[3] and Sette[4]

$$T=0.05V/-\sum s_n\log(1-\alpha_n). \quad (4)$$

These formulas are of limited scope as Sabine's is not accurate for high values of α and the other gives zero reverberation time if any portion of the surface is completely absorbent. None of the formulas gives any weight to the location of the absorbing material in the room. P. M. Morse has derived equations[5] that do involve the location of the material but their accurate solution is very difficult. For the case of a room with one surface much more highly absorbent than the others an approximate solution is an equation similar to Eyring's (Eq. 2) but with the absorption of the treated surface taken at half of the usual value. The resulting reverberation time is nearly twice that obtained by the other formulas.

In the investigation of these discrepancies it was decided to make a graphical analysis of an acoustically treated large office and compare the results with those derived from the above formulas. To facilitate this analysis it was assumed that the ceiling was totally absorbent and that the walls and floor had a uniform absorption coefficient of less than unity.

METHOD OF GRAPHICAL ANALYSIS

The most convenient and accurate graphical means of determining the acoustical character-

[2] J. Acous. Soc. Am. **1**, 217 (1930).
[3] J. Acous. Soc. Am. **4**, 69 (1932).
[4] J. Acous. Soc. Am. **4**, 193 (1933).
[5] *Vibration and Sound*, Sect. 33.

istics of a room is the method of images used by C. F. Eyring.[2] In this procedure a source of known power (E) is assumed to be in the room and the reflecting walls are imagined replaced by series of images of the source, the power of each image source being

$$E_n = R^n E \tag{5}$$

where $R=(1-\alpha)$ = reflecting coefficient of the surface, n = order of the image.

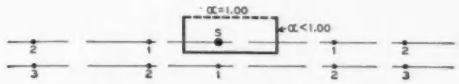


FIG. 1. Elevation view of images of a sound source.

Under steady-state conditions the energy density of the reflected sound (e_p) at any point (P) is the summation of the densities established by all of the image sources.

$$e_p = \sum_{0}^{\infty} E_n/(4\pi d_n{}^2 c), \tag{6}$$

where d_n = distance from the point of observation (P) to the image in cm.

When the source is shut off, all of the image sources instantaneously cease to radiate. Shortly thereafter the end of the wave train from the nearest image reaches the point of observation and the energy density is decreased by the amount of its component. Subsequently the wave trains from increasingly remote images cease and the energy density continues to decrease. Therefore the density at a time "t" seconds after the source has stopped is

$$e_{pt} = \sum_{d=ct}^{\infty} E_n/(4\pi d_n{}^2 c). \tag{7}$$

With this equation and a chart of the location and order of the images it is possible to plot a curve of the sound in the room as a function of time and from it determine the steady-state energy density and the reverberation time.

Charts of the image location for a room with totally absorbent ceiling are shown in Figs. 1 and 2. As the reflection coefficient of the ceiling is zero, the strength of all images above the ceiling level is zero. This confines the images to the two horizontal planes shown in Fig. 1, the one being the plane of the source and the other the plane of the first reflection from the floor. The power of all images in the second plane will be "R" times that of the adjacent image in the first plane. Also, if the source and the point of observation are near the floor, the distances to the adjacent images will be practically equal. Under such conditions the second plane of images may be neglected if the strengths of the images in the first plane are considered to be $(1+R)$ times their actual value. The pair comprising the source and the first order image from the floor may be an exception to this on account of their closeness to the point (P).

Figure 2 is a plan view of the images in the plane of the source, the numbers indicating the order of the images. For simplicity of calculation the area may be divided into zones and all images within a zone considered to be equally distant from the point of observation.

RESULTS

Using this method of analysis, the reverberation time and the energy density were determined for a $33' \times 66'$ room with totally absorbent ceiling and with walls and floor having a uniform absorption coefficient of various values less than unity. These characteristics were also calculated by formulas (1) and (2) for $11'$ and $50'$ ceiling heights. It is important to note that the ceiling height is not a factor in the graphical analysis. This is a logical condition as any sound energy starting in an upward direction will never come down again and the exact moment of its absorption is of no interest. As all of the reverberation

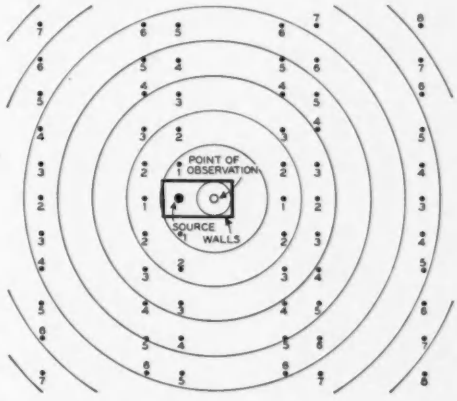


FIG. 2. Plan view of images of a sound source in the plane of the source.

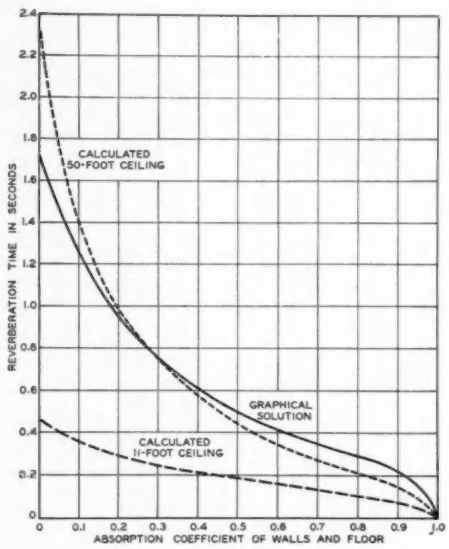


FIG. 3. Reverberation time of a 33′×66′ room with totally absorbent ceiling.

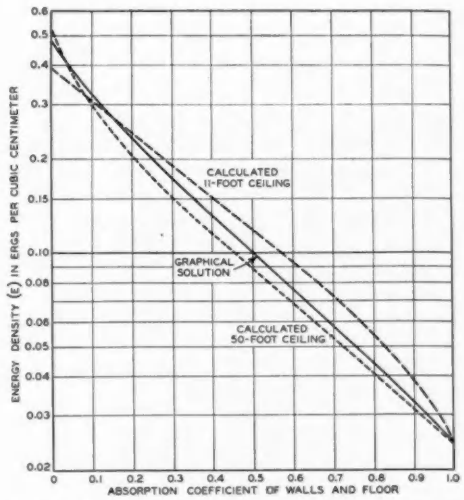

ENERGY DENSITY (ε) IN ERGS PER CUBIC CENTIMETER
CALCULATED 11-FOOT CEILING
GRAPHICAL SOLUTION
CALCULATED 50-FOOT CEILING
ABSORPTION COEFFICIENT OF WALLS AND FLOOR

FIG. 4. Energy density at a point in a 33′×66′ room with totally absorbent ceiling.

time formulas involve the volume of the room they are fundamentally in error in this respect.

The reverberation times as functions of the absorption coefficient of the walls and floor are shown in Fig. 3. The calculated values for the 50′ ceiling height are in reasonably good agreement with those obtained by the graphical solution but the calculated values for an 11′ ceiling height are much too low.

Corresponding curves of energy density *vs.* absorption coefficient are shown in Fig. 4. The agreement in this characteristic is much better, especially at the low absorption coefficients usually encountered in practical cases. The graphical curve falls between the calculated ones but is closer to that for a 50′ ceiling than for an 11′ one.

In the above graphical solutions it was assumed that all of the walls were perpendicular, under which condition the sound will be reflected back and forth between them indefinitely. If one or more of the walls are out of plumb, the image sources will not be confined to the two horizontal planes but will be progressively raised or lowered as they become increasingly remote. As a result, a larger portion of the sound energy will strike the ceiling and be absorbed by it, thus reducing the effective power of the high order images below the value indicated by Eq. (5). This results in a shorter reverberation time and reduces the difference between the graphical and calculated values for low ceiling heights. As these high order components are of minor importance when the sound source is operating, their reduction reduces the steady-state intensity by only a small amount. Furniture or wall irregularities, by producing diffusion, will have the same effect as tilted walls.[6]

The measurements of reverberation time and intensity that were made in offices with acoustically treated ceilings give values in good agreement with those predicted graphically. In one case a room with approximately the same dimensions as the one analyzed was found to have a reverberation time of 0.96 second at 1000 cycles although the time calculated by Eyring's formula was 0.57 second. In another more nearly cubical room the measured and calculated values were 0.83 and 0.56, respectively. If Sette's formula (Eq. (4)) had been used, the calculated times would have been still shorter, and if Morse's is used, they are too long. The validity of the energy density Eq. (1) has been checked frequently by comparison of measured and calculated values of 6 to 7 db noise reduction produced by the installation of a sound absorbent ceiling.

CONCLUSIONS

From the foregoing analysis it is apparent that a room having a highly absorbent ceiling

[6] These same factors impair the accuracy of Morse's analysis.

may have a reverberation time much longer than that calculated by the usual formulas. For similar reasons the total absorption in such rooms cannot be calculated from the reverberation time. In general the error in calculation of auditoriums will be less than that of low ceiling office type rooms.

In contrast to the reverberation formula, that for energy density gives good agreement between calculated and actual values. The reduction in energy density may therefore be used as a measure of the additional absorption resulting from the installation of absorbing material. Unfortunately it is seldom practical for field measurements of this energy reduction to have an accuracy better than ± 0.5 db with a resulting uncertainty in the absorption coefficient of the order of ± 10 to 20 percent.

Three specific conclusions may, therefore, be made from the above analysis. They are (1) that the reverberation time formula (Eq. (2)) is usually invalid if the absorption is concentrated on one surface of the room, (2) that the energy density formula (Eq. (1)) is not subject to such a restriction and (3) that under such conditions there is no apparent field method of determining the absorption coefficient that will serve as an accurate check on laboratory measurements.

Absorption Effects in Sound Transmission Measurements

PAUL E. SABINE AND L. G. RAMER
Riverbank Laboratories, Geneva, Illinois

(Received March 22, 1938)

THE proposed tentative standards for measuring sound transmission of partition walls is contained in pamphlet Z/24/38 dated May 12, 1937 of the American Standards Association. The numerical measure of the reduction of sound intensity proposed is Transmission Loss in Decibels, or, more briefly, Transmission Loss. "This is defined as ten times the logarithm of the base 10 of the reciprocal of the transmittivity τ. The latter is defined as the ratio of the rate of transmitted sound energy away from the partition to the rate of flow of incident sound energy, measurements to be made with a diffuse distribution of both the incident and transmitted sound."

It is obvious that this definition goes back to the picture of the behavior of sound in closed spaces according to the reverberation theory. Sound considered simply as energy is supposed to strike the partition at random incidence and a fraction τ, the transmittivity, is supposed to leave the partition on the further side. One recognizes that even if the conditions assumed in the definitions could be realized in practice the difficulty of measuring the quantities involved is very great. Microphones or other sound measuring devices record either alternating pressure or particle velocity, and tell us nothing of energy flow. Furthermore, it appears that the rate of flow of sound energy to the partition on the incident side will depend upon the absorptivity of the partition. The net rate of flow away from the further side is a function of the total equivalent absorption of the receiving room. Thus in a nonabsorbent space the net flow of energy away from the partition would be very slight since the energy reflected back from the boundary surfaces to the transmitting partition would be almost equal to the energy radiated from the partition. Into a highly absorbent room the flow from the partition would approach that radiated into free space. It appears therefore that the transmission loss, as defined, is not independent of acoustic conditions on the two sides.

A more nearly factual representation of the phenomenon of transmission between adjacent rooms is that of two vibrating bodies of air dynamically coupled by the intervening partition. Each room has its own series of natural frequencies and energy is fed from one to the other by means of the vibration set up in the partition by the alternating pressure exerted by the sound.

It follows therefore that at any fixed frequency the relation between the steady state intensities measured at two points on opposite sides of the partition will depend upon other factors in addition to the sound-insulating properties of the wall itself. Under these conditions a shift in microphone positions relative to the stationary wave patterns might well produce differences in the intensity ratio as great as that due to the partition itself.

In order therefore to treat the problem from the simple energy point of view, it is necessary that local variations due to the standing wave pattern either be eliminated or else that measurements be made in such a way that these local variations are averaged out. The latter can be accomplished by using warble frequency test tones and a long-period meter whose readings will correspond to average values of the meter over the warble frequency range. If measurements are made in this way at sufficiently frequent intervals and if proper allowance be made for the effect of absorption upon the intensity of sound in the receiving room, numerical values can be arrived at which should be a measure of the intrinsic sound insulating properties of the partition.

Considering the problem in this way, we may make the assumptions, (a) that the average sound energy density E_2 in the receiving room is proportional to E_0 the average sound energy density adjacent to the partition in the source room, (b) that E_2 varies directly as the area A of the intervening partition, and (c) that E_2 is inversely proportional to the equivalent absorp-

tion a of the receiving room. On these assumptions, we may write

$$E_2 = \tau E_0 A / a_2. \quad (1)$$

In this equation, τ is a proportionality factor whose value is independent of the room conditions on the two sides and presumably depends only upon the sound insulating properties of the intervening partition. Expressed in decibels, Eq. (1) becomes

$$10 \log (1/\tau) = 10\left(\log \frac{E_0}{E_2} \cdot \frac{A}{a_2}\right). \quad (2)$$

It will be noted that τ in the foregoing is defined not as the "transmittivity" of the partition, but simply as a proportionality factor in the equation giving the relationship between measurable quantities of Eq. (1). The transmission loss may be defined as 10 log $1/\tau$, τ being defined as indicated.

Transmission loss measurements therefore entail the determination of the ratio between the sound energy density on the source side near the partition and the average sound energy density in the receiving room, together with a knowledge of the total equivalent absorption of the receiving room. It is the purpose of the present paper to show that the value of the transmission loss for a given partition is independent of a_2, the total equivalent absorption of the receiving room.

Experimental Procedure

The experimental setup for testing the foregoing conclusion is shown in Fig. 1. The loudspeaker source of sound, placed in the sound chamber was driven by the amplified current from a beat frequency oscillator. Warble tones with a warble range of ±20 cycles and a warble frequency of 6 per second were used. Two 6-inch permanent magnet dynamic speakers were used as pickup microphones, on the opposite sides of a sound insulating door 3×7 feet placed in the vestibule opening off of the sound chamber. The large vanes ordinarily used in the absorption measurement in the sound chamber were kept rotating during the measurements. The receiving room was 7.87×6.0×4.14 feet. A few of the natural frequencies computed by the Rayleigh formula,

$$n = c/2[p^2/l^2 + q^2/w^2 + r^2/n^2]^{\frac{1}{2}}$$

are as follows:

p	1	0	1	0	2	1	1
q	0	1	1	0	0	0	1
r	0	0	0	1	0	1	1
n	71	93	112	134	142	154	179.

Obviously the ±20-cycle warble in the test tone covers the range between natural frequencies

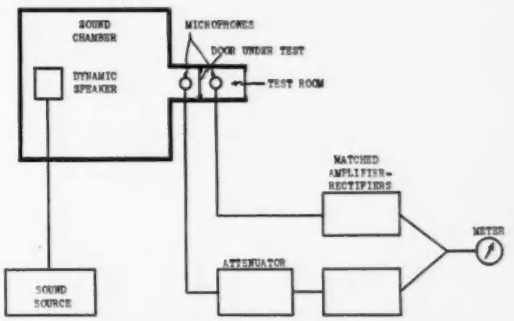


FIG. 1. Experimental arrangement.

thus reducing the otherwise very large effects resulting from resonance in the receiving room. Experiment showed that the combination of the warble tone and revolving vanes averaged out the effect of the standing wave pattern on the transmitting side so that the measurements were practically independent of the microphone positions. The microphones were maintained at fixed positions 22 inches from the door on either side.

The current generated in each of the two microphones was fed into one of a pair of matched amplifiers. An attenuator was placed in the circuit of the microphone on the source side, care being taken of course to secure the necessary impedance match at its terminals. The rectified outputs of the amplifiers were fed in opposite directions through a high sensitivity long-period galvanometer which served as a null indicator for equal outputs from the amplifiers. The measurement then consisted simply in supplying attenuation in the source-room pickup circuit sufficient to balance the acoustical attenuation produced by the partition. Error due to a difference in sensitivities of the microphones, or of amplification in the amplifiers was eliminated by interchanging microphones and amplifiers and taking the average value of the balanc-

ing attenuation as equal to the difference of intensity levels on the two sides of the partition. Measurements were made at half-tone intervals covering the range from 128 to 2000 cycles, and at half-octave intervals from 2000 to 4000 cycles. Measurements were made with the equivalent absorption of the receiving room varied from 7.1 units, the condition with no absorption except that of the bare room itself, to a heavily padded condition with a total average absorption of 131 units. Fig. 2 shows the reduction in decibels in intensity level for the two extreme conditions. The fact that the two curves are so nearly parallel indicates that the marked variations with frequency are to be ascribed to resonances in the partition rather than to receiving room resonance. Similar measurements were made with three intermediate degrees of room damping. In all cases, the reduction-frequency curves showed the same characteristics, but with the average ordinate increasing linearly with the total average absorption as shown by the Table I.

The foregoing bears out the conclusion that transmission loss, defined by the equation,

$$\text{T.L.} = 10 \log (E_0/E_2) \cdot A/a_2,$$

is independent of the total equivalent absorption if we take values averaged over an extended frequency range.

It further shows that a simple measurement of the difference of intensity levels on the two sides of a partition does not, in general, give the transmission loss, since both the area of the intervening wall and the total equivalent absorption of the receiving room enter into the difference of intensity levels. This fact is often ignored in field measurements of the insulating effectiveness of walls, doors and windows. The assumption as to the effect of the area of sound transmitting surface still has to be experimentally verified.

TABLE I.

a_2	10 log a_2	10 log E_0/E_2	10 (log E_0/E_2 − log a_2)	T.L.
7.1	8.5	33.2	24.7	37.9
25.1	14.0	38.8	24.8	38.0
27.9	14.5	39.2	24.7	37.9
81.1	19.1	44.3	25.2	38.4
131.0	21.2	45.8	24.6	37.8

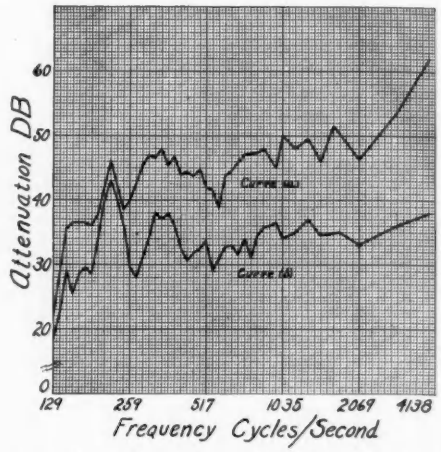


FIG. 2. Attenuation. Curve (*a*), Receiving room absorption 131 units; Curve (*b*), Receiving room absorption 7.1 units.

In view of the foregoing it is suggested that the term "transmission loss" be defined not in terms of "transmittivity," a hypothetical quantity which is not directly measurable, but by the equation

$$\text{T.L.} = 10 \log (E_0/E_2) \cdot A/a_2$$

or more directly as "difference in intensity level minus 10 logarithm of the ratio of the total equivalent absorption of the receiving room to the transmitting area." "Transmittivity," if there is any need for the term, can then be defined as the reciprocal of the anti-logarithm of 1/10 the transmission loss.

Absolute Sound Measurements in Liquids*

ELIAS KLEIN
Naval Research Laboratory, Washington, D. C.
(Received May 23, 1938)

The fundamental problem of absolute sound measurements in liquids is discussed. Regarding acoustic determinations, reference is made to certain inherent differences between air and water. An outline is presented of three methods now available for sound determination. A modified radiation pressure apparatus is described which permits a dual check of the basic measurements involved, and at the same time makes possible microphone calibration for secondary standards.

INTRODUCTION

ABOUT eleven years ago, Professor R. W. Wood demonstrated the effects of supersonics, as transmitted through liquids. One pertinent and impressive experiment is worth recalling. By means of intense high frequency sound waves he caused oil to emulsify and spout from a glass vessel. He also told us of paralyzing or incapacitating fish, frogs and mice by exposing them to this form of radiation. Wood and Loomis[1] described many other spectacular and curious phenomena associated with supersonics which at that time seemed primarily of academic interest. Today, the propagation of sound in liquids has become widespread in its applications. Aside from those who are concerned with subaqueous acoustics as related to navigation, a large number of investigators are extending the uses of high frequency vibration in liquids to many industries. If in the place of the novel oil spray from a beaker which Professor Wood showed us, you can visualize the frothing and foaming of a large vat of beer as a result of sound; or, the sterilization of milk[2] bacteria instead of a goldfish wiggling under the influence of supersonics, you will have the transformed picture of a scientific curiosity turned to commonplace utility. Moreover, by the aid of high frequency sound, the structure of liquids as well as many of their physical properties are now being determined.

Due to the diversified applications of acoustics in liquids, the vibration frequency range has been extended into the megacycle region; while the sound intensities utilized are limited only by the vapor pressure of the medium in which the sound is generated. But the known methods for sound measurements in this field of activity have not kept pace with its rapid growth. Of course, the progress in aerial acoustics has aided materially in the development of a measuring technique in liquids. However, the problems of establishing sound standards in gases and liquids are not at all parallel, because of certain characteristic dissimilarities in the media. More specifically, air and water require distinct methods and often involve different forms of apparatus for the determination of absolute sound in these fluids. To illustrate this point, let me cite two outstanding acoustic differences between air and water.

Absorption

To my knowledge there has not yet been developed a substance which definitely absorbs sound in water. A tank whose walls would dissipate about 99 percent of the energy incident upon them (something comparable to the "dead" room here at the Bureau of Standards) should be of tremendous help to those investigating underwater sound. In the absence of such material, and until the equivalent open sea conditions are available, all laboratory measurements must be made in some form of standing wave system. Furthermore, experience has shown that for the determination of absolute sound in liquids, it is more desirable to perfect and simplify the stationary wave pattern than to attempt to annihilate it. That is one direction in which the development of laboratory technique is tending. It is quite different from "dead" room studies in air.

* Paper presented at the Symposium on Acoustic Standards before the Acoustical Society of America, at the National Bureau of Standards May 2–3, 1938.

[1] Wood & Loomis, Phil. Mag. **4**, 417 (1927).

[2] N. Gaines, Physics **3**, 209 (1932).

Cavitation

Another point of departure between these fluids is cavitation. Unlike air, water (or any other liquid) cannot be made to transmit very intense sounds because it breaks down when a sound generator in the liquid medium exceeds a given amplitude. The phenomenon is known as cavitation and is often confused with the expulsion of absorbed gases from a liquid when the latter is subjected to strong vibrations. Cavitation occurs in an ordinary gas-free liquid if the rarefaction in the liquid is greater than the existing static pressure at that level. In other words, the liquid is unable to follow the amplitude excursion of the vibrating piston or membrane, and the liquid is stretched to the point of rupture. Thus the intensity of sound which may be generated in a liquid is limited by the vapor pressure of the liquid and the immersion depth of the generator. The maximum energy per square centimeter theoretically attainable is[3]

$$E=(p_0-p_v)^2/2\rho c^2,$$

where E is the energy density, p_0 the hydrostatic pressure, p_v the vapor pressure, and ρc the acoustic resistance of the liquid.

This limiting condition of transmissibility in liquids has also its advantages. For example, it may be used in standardizing a sound generator. The latter is excited until cavitation is reached. Then, from the physical constants of the liquid and the depth of the source, the radiated energy density may be estimated.

Another fundamental difference between gases and liquids has been noted by Lucas and Biquard,[4] who report that in liquids there has been observed no dispersion in the velocity of sound as is the case in gases.

These and other characteristic dissimilarities of gases and liquids make the development of absolute sound measurements in these fluids only remotely analogous.

Standard Methods

In view of the large variety of uses for sound in liquids, it is difficult to suggest sufficiently diversified measuring standards. Nevertheless, the following three methods, arbitrarily selected, are potentially suitable as acoustic standards in liquids for industrial applications as well as for other uses. The general interest in sound standards is obvious. Quantitative measurements are indispensible to the acoustical engineer and designer of sound equipment. Specifications and acceptance tests are more significant when referred to a basic standard. At present there is no accepted standard for acoustics in liquids.

Diffraction of light by sound waves

One approach to absolute sound determinations is by means of optical diffraction. For structural studies of and absorption measurements in liquid, this method is very promising. It has been particularly effective in the acoustic frequency band between 0.5 and 80 megacycles. The basic idea of the phenomenon is that a liquid carrying high frequency elastic waves exhibits certain properties of an optical grating. Brillouin[5] predicted the general effect theoretically. It was observed experimentally by Debye and Sears[6] in this country and Lucas and Biquard[7] in France. A simple laboratory set-up of the apparatus as shown in Fig. 1, is before you on the table. A rectangular glass cell with plane parallel sides is filled with the liquid under investigation. Supersonic waves generated by the quartz crystal at the top produce in the liquid a series of planes in which the density varies periodically from maximum to minimum. From the left a parallel beam of monochromatic light is made to pass through the liquid, perpendicular to the sound rays. The emergent beam of light is broken up as if it had come from a diffraction grating. In the focal plane of the

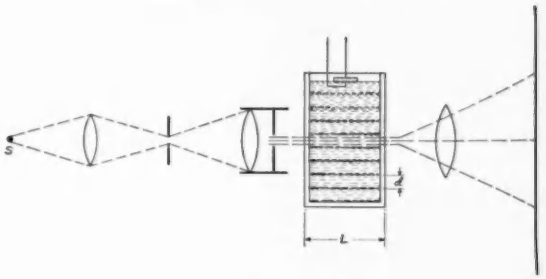
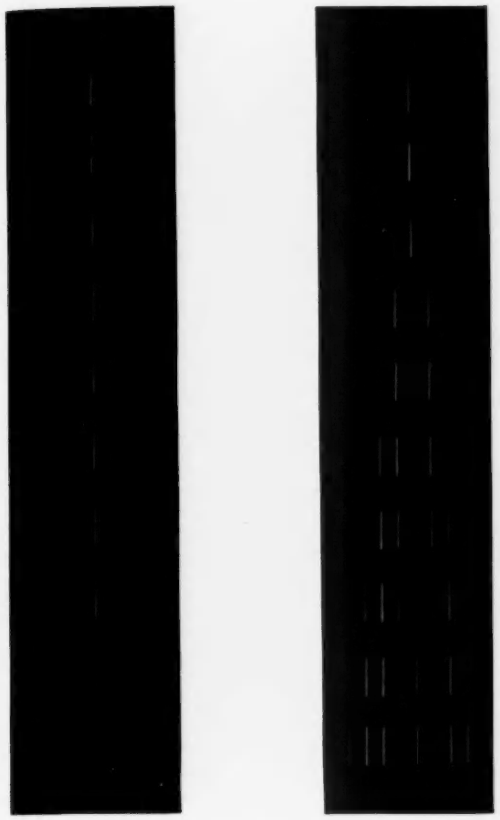


Fig. 1.

[3] Boyle & Taylor, Phys. Rev. **27**, 518 (1926).
[4] Lucas & Biquard, Trans. Far. Soc. **33**, 131 (1937).
[5] L. Brillouin, Ann. de Physique **17**, 103 (1921).
[6] Debye & Sears, Nat. Acad. Sci. **18**, 409 (1932).
[7] Lucas & Biquard, Comptes. Rend. **194** (1932).



(a) (b)

FIG. 2.

lens (to the right) may be observed a series of images of the slit S. Moreover, experiment shows that for normal incidence the transmitted beam obeys the simple grating law, namely $N\lambda = d \sin \theta$. The grating space d is the wave-length of sound in the liquid and determines the spacing of the slit images. Optically, d is a measure of the periodic changes in the density of the liquid. The number of orders depends upon the intensity of sound.

So much for the essential features of the experimental observations. Of course, oblique incidence produces corresponding diffraction orders for either progressive or stationary wave systems. Fig. 2 is reproduced* to show the possibilities of this method. The results from this type of measurement indicate that the relative intensities of the light amongst the various diffraction orders vary with the length of the cell, the strength of the sound field and the wave-length of the incident light. In other words, the sound intensity at any point in the liquid may be evaluated in terms of photometric determinations of the lines in the different orders, provided a definite relation exists between the two intensities. One expression[8] showing a relationship between the intensity of the mth and nth orders is:

$$I_m/I_n = (J_m^2(V))/(J_n^2(V)).$$

J_m and J_n are Bessels functions of the corresponding order. The argument V is given by:

$$V = (2\pi L \delta n)/\lambda,$$

where L is the length of the sound cell through which light is passing, δn is the maximum variation in refractive index, and λ the wave-length of the incident light.

To establish this relationship, several theories have been evolved. At present, the mathematical expressions developed by Raman and Nath[9] for the intensities of light in the various orders show a reasonable agreement with experimental results of numerous investigators.

Incidentally this diffraction method has been used quite successfully[10] to determine the velocity of sound in liquids at high frequencies. For this purpose, only the direction of the diffraction beam need be measured. The velocity of a large number of organic liquids has thus been recorded.

To designate briefly the salient features of the diffraction method for acoustic standardization, one may say:

(a) It makes possible the evaluation of the existing sound field as no other method does since it required no detector or obstacle in the sound medium to disturb the wave or abstract energy from it.

(b) It is primarily suited to very high frequencies.

(c) It involves photometric measurements of some extremely faint lines which reduce the accuracy of the final determination.

* Published with the kind permission of the author, F. H. Sanders, Can. J. Res. **14**, 158 (1936).

[8] Raman & Nath, Proc. Ind. Acad. Sci. **2A**, 410 (1935).

[9] Raman & Nath, Proc. Ind. Acad. Sci. **3A**, 459 (1936); F. H. Sanders, Nature **138**, 285 (1936); Can. J. Res. **14**, 158 (1936).

[10] S. Parthasarathy, Proc. Ind. Acad. Sci. **4A**, 213 (1936).

(d) Existing theory and experiments are not yet sufficiently in accord to rate this method as a primary sound standard. However, owing to the simplicity and immense possibilities of investigating a sound field by an optical arrangement, this system of measurement provides a definite basis of comparison with other absolute methods.

The Rayleigh disk

Another means for absolute measurement of sound in liquids is by the use of a Rayleigh disk. This method is well known to all of you. With the Rayleigh disk as a standard in air, various microphones have been calibrated in open field or "dead" room chambers, as well as in confined vessels amidst standing wave systems. Its advantages and limitations as applied to aerial acoustics have been adequately discussed by many workers. However, to employ this instrument for absolute measurements in liquids, König's original theory requires additional interpretation. The classical theory demands that the disk be thin, perfectly rigid, small compared to the sound wave, and except for rotation, that it remain stationary relative to the fluid in which it is immersed. For audiosounds in air these conditions are generally fulfilled. Whereas, in a liquid like water, the inertia effect of the medium upon the disk, the likelihood of sound transmission through the disk, and the setting up of flexural vibrations in the disk are among the factors which accentuate the divergence between theory and experiment.

A correction to the theory of the Rayleigh disk as applied to sound measurements in water was recently made by A. B. Wood.[11] He obtained the following modified expression for the turning moment acting upon the disk:

$$T=4/3\rho a^3 V_w^2(1-\beta)^2 \sin 2\theta.$$

In which $\beta = V_d/V_w$,

where V_d is the r.m.s. velocity of the disk and V_w is the r.m.s. velocity of the water. Wood also demonstrated the application of the theory by a series of experiments which showed the possibilities of the Rayleigh disk as an acoustic instrument in water up to 20,000 cycles. The disks used and the frequencies explored by Wood were easily within the rule that the shortest wave-length in the medium shall be at least five times the diameter of the disk. In fact, the greater velocity of propagation in water compared with air favors the use of the disk in water to the limits of audible frequencies, were it not for other difficulties to which the disk is subject as its natural vibration periods are approached.

Accordingly, it should be observed that although the Rayleigh disk is a very desirable and practical standard for sound intensity measurements in air, it is not ideally suited for absolute determinations in liquids. Its useful frequency range is limited because the disk is not rigid relative to the liquid; and, its sensitivity is quite low. Primarily a liquid transmits pressure more readily than displacement or velocity. Consequently, any underwater sound measuring device should preferably be of the pressure type. King[12] who has made a theoretical investigation of the Rayleigh disk as an acoustic instrument in water, showed that the minimum sound energy required to actuate a given disk in water is about 35 decibels greater than is necessary to produce a similar deflection on the same disk in air. This further emphasizes the need for careful differentiation in setting up acoustic standards for use in gases and in liquids.

Radiation pressure

The third method for studying quantitatively the sound field in liquids is by the effects of radiation pressure. This phenomenon makes possible the determination of the energy density at a given point in the medium. Any device operating on this principle is known as a radiometer. Like the Rayleigh disk, the radiometer measures a rectified component of the vibratory energy incident upon it in terms of mechanical units. The distinction is merely in the form of the energy imparted to the instrument. Kinetic energy produces the torque in the former, while the latter responds mainly to the potential energy of the medium. Whether a velocity meter or a pressure indicator both are in effect second order approximations.

[11] A. B. Wood, Proc. Phys. Soc. **47**, 262 (1935).

[12] L. V. King, Proc. Roy. Soc. **153**, 17 (1935).

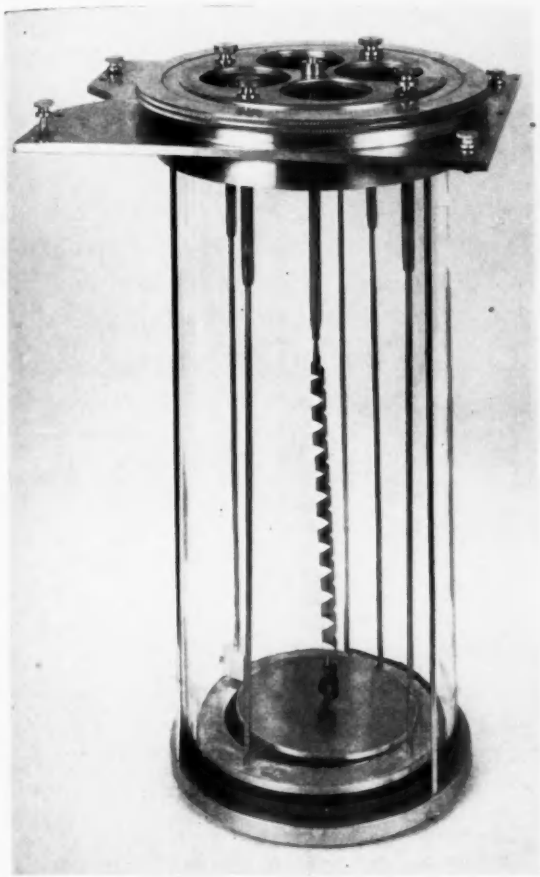


FIG. 3.

For relative measurements any type of radiometer will approximate the sound field intensity at a point in the liquid. Torsion pendula and beam balances have been used for such purposes with moderate success. The experiments of Boyle[13] and his co-workers who employed disk radiometers, and the methods of "weighing" radiation pressure as used by Sörensen[14] and others illustrate comparative results. In attempting absolute measurements by means of torsion balances, the general character of the radiation field and its modification by the detector must be taken into account before the results can be correlated even with elementary analysis. The very nature of radiation pressure presupposes the existence of a nonlinear force in the medium.

[13] Boyle & Froman, Can. J. Res. **1**, 405 (1929), and other papers.

[14] Chr. Sörensen, Ann. d. Physik **26**, 121 (1936) and **27**, 70 (1936).

It was shown by E. Fubini-Ghiron[15] that, although a generator may emit sound in a perfectly sinusoidal manner, the wave in its progress through the medium is gradually deformed and distorted. Beyond a distance X from the source,

$$X=2C^2/((\gamma+1)AW^2)$$

the wave front resembles ocean breakers approaching a shore.

Therefore, in the apparatus we are using for absolute measurement, an attempt has been made to reduce the unavoidable approximations to a point which is consistent with the validity of simple theory. A spherical torsion pendulum has been adopted because of its geometrical simplicity and because the diffraction theory about a sphere is well known. Indeed, the theory for acoustic radiation pressure on disks has recently been completed;[16] and, it develops that a small spherical radiometer is about 20 times as sensitive as a disk of the same radius when immersed in a progressive wave. It is four times as sensitive in a stationary field. The standing wave system in the specially designed cylindrical containers assures a minimum transfer of energy to the surrounding medium, and at the same time makes possible the exploration of the sound field on the axis of the cylinder quite accurately.

A complete description of this type of apparatus together with some results of preliminary measurements is given in the last issue of this journal.[17] Reference is made there to another form of acoustic radiation pressure device which was demonstrated to the Society a year ago. It consists of a special helical spring supporting a disk at its free end. Fig. 3 shows the spring with one of the reflectors. The principle of its operation is interesting. Sound pressure generated by a beam-source located at the base of the cylinder, produces an upward thrust upon the piston and lifts it against gravity, thus reducing the extension of the spring. And, the nature of the latter is such that a very small elongation or contraction of its length gives the free end a relatively large axial rotation, as if it tended to unwind or wind-up. In other words, a tiny linear displacement is transformed and

[15] E. Fubini-Ghiron, A. F., **4**, 530 (1935); **6**, 640 (1937).

[16] L. V. King, Proc. Roy. Soc. **153**, 1 (1935).

[17] Elias Klein, J. Acous. Soc. Am. **9**, 312 (1938).

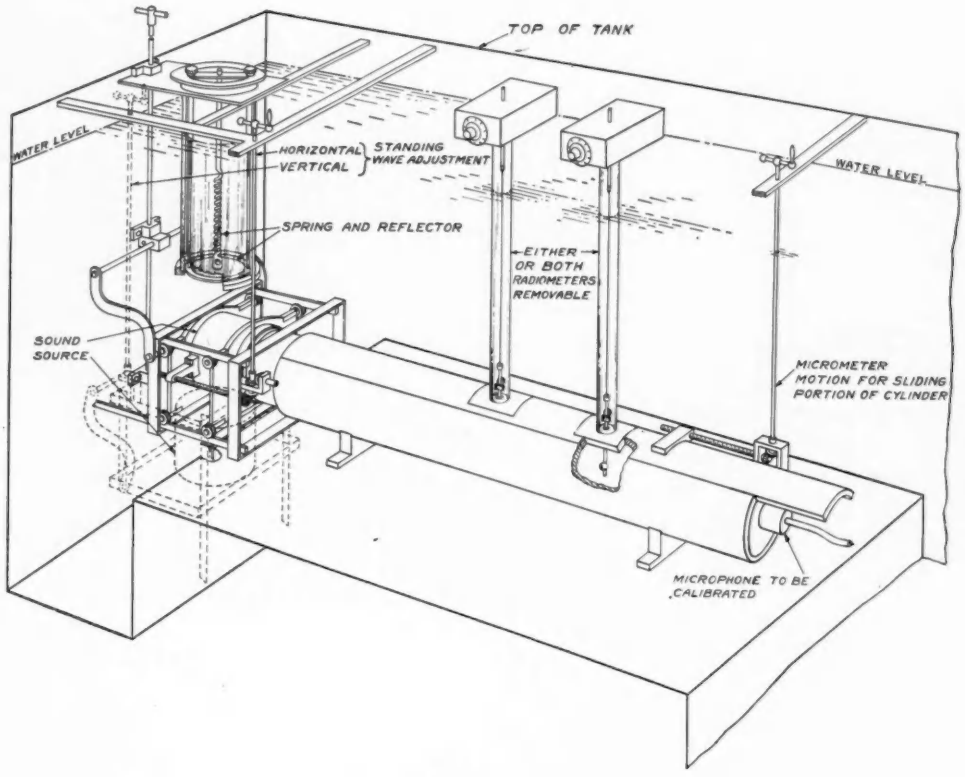


FIG. 4.

amplified into a huge angular displacement. It is quite sensitive, though rugged; with the loading as shown, its torsion constant in oil is 10 dynes-cm/degree. It is comparatively free from temperature drift or creep; and, repeats settings remarkably well.

As a sound-pressure meter, this "weighing" device differs from the spherical pendulum in that it involves no diffraction. Here the beam of parallel rays is not disturbed or scattered, but is reflected as a whole by a piston plate whose surface is normal to the wave propagation. The piston with very small clearances is movable in a rigid guard-ring. Total normal reflection is assumed at the ring and piston. Consequently, by adjusting the distance between the sound generator and reflecting surfaces, a perfect standing wave system is produced. Under such conditions the radiation pressure actuating the disk in water is twice the energy density in the medium, or $R_p = 2E$.

Thus by combining these two instruments it is possible to determine the sound output of a generator by independent methods under similar conditions. Fig. 4 shows the manner in which this is accomplished. It also indicates the provision made for studying the interaction and scattering effects of one spherical pendulum upon another. The center of each sphere remains on the axis of the cylinder which is substantially the axis of the sound beam. The positions of the pendula are interchangeable. Each can be removed without affecting the interior of the cylinder.

Analysis shows that the greater the density of the sphere compared with the medium, the closer the simple radiation pressure theory approximates experimental results. Consequently, both radiometers shown have a density of over 21 g/cm^3. They consist of a platinum-iridium alloy. As a result of this large density relative to water, the ratio ρ water/ρ sphere becomes negligible in evaluating the sound pressure for routine calibrations.

Without, at present, going into the details of measurements or the procedures followed to re-

duce them to the required pressures, a few results may be mentioned. Several crystal microphones have been calibrated by means of this set-up, and the results compare favorably with some check measurements made on the same crystals in the "dead" room at the National Bureau of Standards. This cooperative work of calibrating crystal microphones in air and in water is being continued.

Summary and Conclusion

In recent years there have been devised many methods for estimating the sound field intensities in fluids. Owing to certain inherent differences between gases and liquids, the latter often involve particular measuring apparatus. In spite of the tremendous electroacoustic developments, the absolute instruments for liquids here discussed depend upon optical and mechanical phenomena rather than electrical. Light diffraction is suitable for structure and absorption studies in liquids at high frequencies. For chemical engineering and marine applications the frequency range extends from the audible to thousands of kilocycles where the Rayleigh disk and the radiometer may be employed. It is believed that each of the methods mentioned has a definite field of usefulness and that each can be made to overlap another for comparative results. Without attempting to assess values to the various types of acoustic standards in liquids, the author's experience in underwater investigations leads him to the preference of radiometers over any other form of absolute instruments.

In the standardizing work, two distinct radiometric devices are incorporated in one vessel so as to determine alternately the radiation pressure developed by a given source under identical conditions. Every effort is made to maintain plane and stationary wave systems in the sound fields under investigation. These circumstances lend themselves more readily to the calibration of microphones which function as secondary standards. The primary instruments are made reasonably sensitive in order to avoid large amplitudes and the anomalies pertaining thereto. This makes for the correlation of simplified theory with actual experiment, the concordance of which will be established only by sufficient observational data.

Theory of the Chromatic Stroboscope

ROBERT W. YOUNG AND ALLEN LOOMIS
C. G. Conn, Ltd., Elkhart, Indiana

INTRODUCTION

SOUND measurements of many kinds are facilitated by the use of ratios, or their logarithms. As a measure of sound intensity level the decibel has become familiar, some suitable reference intensity being expressed or implied. Likewise a frequency encountered in music is conveniently expressed as a number of semitones, or a fraction thereof, from a standard reference frequency.

In this paper is given the basis of a method for the precise instrumental measurement of the logarithm of a ratio. This method was originally conceived by one of us in the form embodied in the chromatic stroboscope.[1] Improvements[2] in this device, described below, have made possible a portable instrument suitable for general use.

OPERATION OF THE CHROMATIC STROBOSCOPE

The stroboscopic principle is employed to compare the unknown frequency of a sound to the frequency with which the dark segments of a rotating patterned disk pass a given point. The convenience of a stroboscope for such comparison has been recognized for some time,[3, 4] but the present device is especially adapted to musical use by having twelve disks rotated at speeds corresponding to the frequencies of notes in a chromatic octave of the equally tempered scale. From this chromatic arrangement the name of the apparatus is derived. Furthermore, deviation from standard tuning may be read with precision, following the simple manipulation of a single knob.

Figure 1 shows the stroboscope unit on the left. The twelve disks, partially covered by a mask, are arranged for identification in the relative position of the black and white keys of the piano keyboard, i.e., the five "black" keys above and the seven "white" below. The chromatic octave is thus represented, starting with C at the lower left. Such an arrangement should be of convenience in grouping any indicators used in connection with frequency measurements in terms of musical intervals.

Sound picked up by a microphone causes these disks to be illuminated intermittently at a rate equal to that of the frequency of the sound. If this happens to be one of the frequencies of the standard tempered scale (to which the piano nominally is tuned) a stroboscopically stationary pattern appears in one of the windows in the mask. If the unknown frequency be increased slightly the pattern will appear to move toward the right, and similarly a lowered frequency will be indicated by motion toward the left. This test can be applied over essentially the entire range of the piano keyboard without any adjustment or delay between observations, since all twelve disks are illuminated at the same time.

It is also possible to read how much the frequency of a musical tone departs from standard tuning. Suppose that a note is being sounded at a frequency greater than standard (i.e., sharp), so that the appropriate pattern appears to move toward the right. By a simple adjustment the speeds of all the rotating disks can be varied simultaneously over a small range. By increasing the speed of the disk slightly it may be thus brought to synchronism with the intermittent illumination (and consequently the unknown frequency), a fact evidenced by the now stationary pattern. An indicator connected with the speed adjusting element shows how much this frequency is above standard, expressed as a fraction of the equally tempered semitone, the unit being the hundredth of a semitone, called the "cent."

RATIOS REQUIRED

Frequency ratios, particularly large musical intervals, are commonly expressed in octaves, where the octave is defined as the ratio 2/1.

[1] O. L. Railsback, "A Chromatic Stroboscope," J. Acous. Soc. Am. **9**, 37–42 (1937).
[2] The principal features were described in a paper "An Improved Chromatic Stroboscope" presented by the authors before the Acoustical Society, May 4, 1937.
[3] C. E. Seashore, "A Voice Tonascope," Univ. of Iowa Stud. in Psychol. **3** (1902); *The Psychology of Musical Talent* (1919), p. 189–200; "The Measurement of Pitch Intonation with the Tonascope," Univ. of Iowa Studies No. 172 (1930).
[4] H. W. Lamson, J. Acous. Soc. Am. **7**, 51–55 (1935).

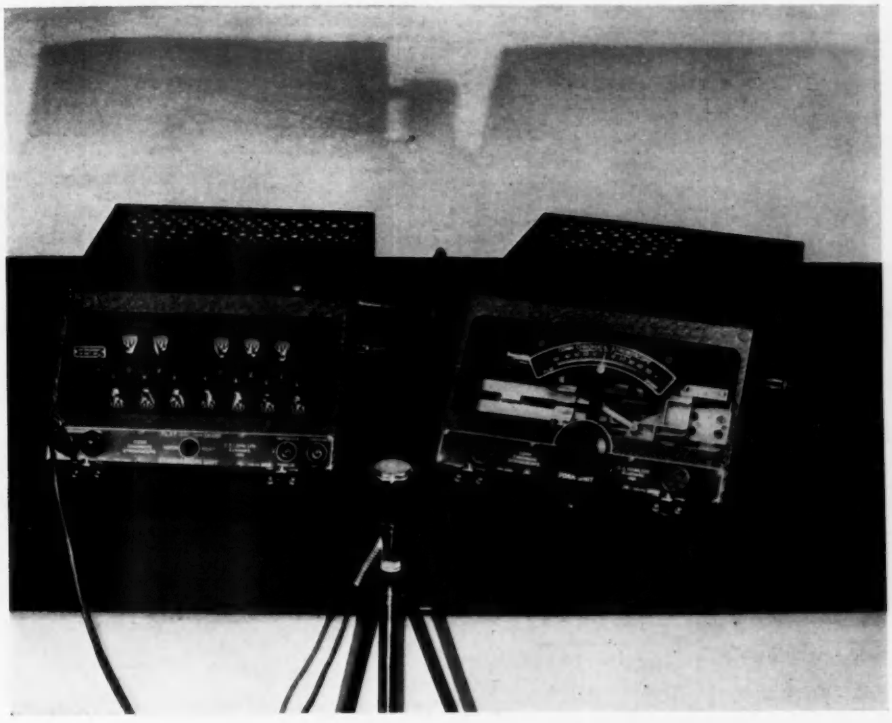


FIG. 1. The complete chromatic stroboscope.

Expressed formally, any two frequencies f and f_0 are separated by a number of octaves, N, given by

$$f/f_0=2^N, \qquad N=\log_2 (f/f_0) \text{ octaves.} \quad (1)$$

The relationship is general in that it is valid for fractional as well as integral numbers of octaves.

Furthermore, for musical purposes the octave is usually divided into twelve equal ratio intervals called equally tempered semitones. Thus the ratio of two frequencies f and f_0 may be expressed as a number of semitones, S, found by

$$f/f_0=2^{S/12}, \qquad S=12\times\log_2 (f/f_0) \text{ semitones.} \quad (2)$$

In like manner the semitone may be divided into cents (100 cents = 1 semitone), the number of cents, C, being given by

$$f/f_0=2^{C/1200}, \qquad C=1200\times\log_2 (f/f_0) \text{ cents.} \quad (3)$$

If we choose f_0 to be some standard reference frequency, say the American Standard Pitch A of 440 cycles/second, we may express any other frequency as a number of octaves, semitones, and cents from this reference frequency. Then

$$f=[440\times 2^N\times 2^{S/12}]\times 2^{C/1200}. \quad (4)$$

For convenience we may limit ourselves to an integral number of octaves, an integral number of semitones not greater than twelve, and a number of cents not greater than 100 (preferably in the range -50 to $+50$). For example, the frequency of a flat C, the highest on the piano, might be described as being 3 octaves, 3 semitones, and -30 cents above the reference A. These numbers substituted in (4) give

$$f=[440\times 2^3\times 2^{3/12}]\times 2^{-30/1200}. \quad (5)$$

Since the part in brackets represents the nominal frequency of the note in question, for many musical purposes it suffices to give only the nominal note and its observed deviation.

Corresponding Elements of the Measuring Apparatus

A null indicator is a desirable feature of many measuring devices, and in the chromatic strobo-

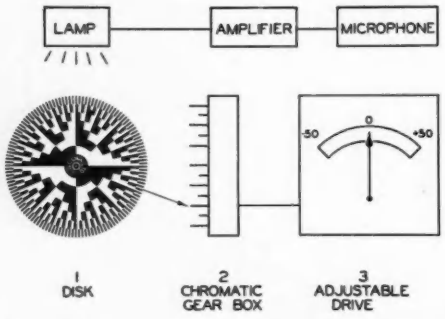


FIG. 2. Elements of the chromatic stroboscope.

scope the appearance of a stroboscopically stationary pattern serves this purpose. Thus our problem becomes that of finding suitable elements corresponding to Eqs. (1), (2), and (3).

1. The disk

As indicated by Eq. (1) our basic standard interval is the octave, the 2/1 ratio. Since the present apparatus is designed to cover the seven-octave range of the piano the disk shown at (1) in Fig. 2 is used. Each ring, progressing radially outward, has twice as many dark segments as the preceding one (2, 4, 8, 16, 32, 64, 128). This disk with its seven rings, when driven at 27.5 r.p.s. (the nominal frequency of the piano's lowest string), is suitable for checking all seven octaves above its fundamental speed of rotation. For example, the 16-segment ring will appear stationary when illuminated by light flashes occurring at 440 per second ($16\times27.5=440$). The disk thus furnishes a basic system of interconnected standards, or reference points, spaced on the 2/1 ratio.

2. The chromatic gear box

Now having in effect divided the frequency range in which we are interested into octaves, we must next find a way of subdividing the octave into intervals of the kind required by Eq. (2). Thus in addition to a disk speed such as given above eleven other speeds must be provided, making 12 in all, to fill in by semitone steps the octave gaps created by the choice of disk pattern.

Figure 2 indicates how the disk might be put on any one of twelve shafts, although for convenience in actual use a disk is permanently attached to each shaft. In accordance with the condition imposed above, successive shafts must rotate at speeds proportional to $2^{1/12}$.

It is of course impossible to obtain precisely the ratio $2^{1/12}=100$ cents by the combination of any finite numbers of gear teeth. However, 89/84 is a practicable approximation which is only slightly greater than $2^{1/12}$. One of us conceived the scheme of balancing this excess by the alternate use of this large ratio and another deficient by about the same amount. The ratio 107/101 was found to fulfill the latter requirement.

Obviously we could compare these quotients directly with the decimal value of $2^{1/12}$. However, since we are interested finally in getting readings in cents it is convenient here to compute the number of cents for the 89/84 and 107/101 ratios, and to compare with the desired theoretical value 100.

Using the relationship of Eq. (3) we have

$$89/84=2^{C/1200}, \qquad C=100.0992 \text{ cents.} \quad (6)$$

It will be seen that this is large by 0.0992 cent.

Likewise

$$107/101=2^{C/1200}, \qquad C=99.9066 \text{ cents.} \quad (7)$$

This is small by 0.0934 cent.

TABLE I. *Intervals obtained with the chromatic gear box. Intervals expressed in cents.*

DISK	GEAR TEETH	Interval	FROM A	THEORETICAL	ERROR
B			+200.0058	+200.0000	+0.0058
	89/84	100.0992			
A♯			+ 99.9066	+100.0000	−0.0934
	107/101	99.9066			
A					0.0000
	89/84	100.0992			
G♯			−100.0992	−100.0000	−0.0992
	107/101	99.9066			
G			−200.0058	−200.0000	−0.0058
	89/84	100.0992			
F♯			−300.1050	−300.0000	−0.1050
	107/101	99.9066			
F			−400.0116	−400.0000	−0.0116
	89/84	100.0992			
E			−500.1108	−500.0000	−0.1108
	107/101	99.9066			
D♯			−600.0174	−600.0000	−0.0174
	89/84	100.0992			
D			−700.1166	−700.0000	−0.1166
	107/101	99.9066			
C♯			−800.0232	−800.0000	−0.0232
	89/84	100.0992			
C			−900.1224	−900.0000	−0.1224

The sum of the two intervals is 200.0058 cents, which is a close approximation to $2^{2/12}$, the error being only 0.0058 cent.

By the successive use of these two ratios in alternation the chromatic gear box pictured in Fig. 2 at (2) is designed to provide disk speeds proportional to the frequencies of the 12 chromatic notes within an octave of the equally tempered scale. Table I gives the intervals concerned, and the calculated deviations from exact equal temperament due to gear approximations. It shows also, for example, that the shaft which carries the A disk carries gears having 84 and 107 teeth, respectively. Likewise the G# shaft has gears of 89 and 101 teeth.

The A disk is driven externally and the other disk speeds are expressed in logarithmic cents with respect to that of the A disk. These figures are given in the fourth column, and are positive or negative depending upon whether the speed in question is greater or less than the A disk speed. The interval error, defined as the theoretical value deducted from the actual, accumulates to a maximum on the C disk of about 1/8 cent. Inasmuch as the cent approximately corresponds to a difference of 0.06 percent the error in frequency measurement due to the gear train is never greater than 0.007 percent.

In finding the number of cents in Eq. (3) ordinary logarithms can of course be used. However, for very accurate computation of the number of cents, C, representing the ratio of two frequencies f and f_0 the following is convenient:

$$C=\frac{2\times 1200}{\log_e 2}\left[\left(\frac{f-f_0}{f+f_0}\right)+\frac{1}{3}\left(\frac{f-f_0}{f+f_0}\right)^3+\frac{1}{5}\left(\frac{f-f_0}{f+f_0}\right)^5\cdots\right]. \quad (8)$$

For the 107/101 case three terms are more than ample:

$$C=3462.46810\times[(288{,}461{,}538+80{,}010+40)\times 10^{-10}]=99.906605 \text{ cents.}$$

This demonstrates that for small intervals the relationship between cents and percentage frequency difference is essentially linear.

The ratio 89/84 has been generally recognized as a good approximation to $2^{1/12}$, but for some reason the 107/101 ratio has escaped general attention. Both may be found by the method of continued fractions,[5] the latter being one of the intermediate convergents. For other gear approximation problems as well, the theory allows one to write down two series of approximations, all of the first being in excess and all of the second in defect of the desired ratio, so that the method of continued fractions provides a convenient way of selecting two ratios, one large and one small, which, used in alternation as described above, will provide a good approximation with relatively small cumulative error. While it is true that the 196/185 ratio used previously[1] (obtained by $49/37\times 24/30$) is nearer $2^{1/12}$, yet without a serious loss in accuracy the present combination of ratios halves the number of gears required, with an attendant reduction in cost, noise, back lash and motive power requirement.

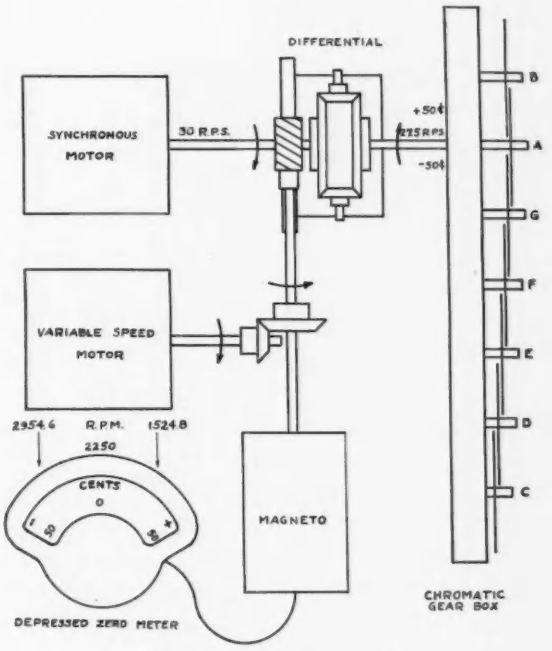


FIG. 3. An adjustable drive.

3. The adjustable drive

By the system just described the frequency range is divided into 7 octaves, each of which is subdivided into 12 semitones. This means that when the A disk is driven at 27.5 r.p.s. there

[5] Chrystal, *A Text-Book of Algebra*, Part II, p. 444.

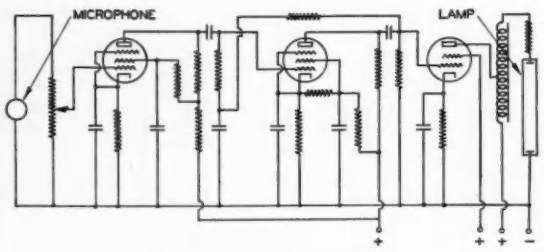


FIG. 4. The flashing amplifier.

will be an appropriate pattern and speed for $7\times12=84$ notes of the chromatic equally tempered scale based on the A of 440 cycles/sec. Furthermore, these 84 reference points are all interconnected, so that if the drive speed of the chromatic gear box be decreased by 1 percent then the 84 frequency standards are each decreased by the same percentage.

Consider any arbitrary frequency within the range of the instrument (32 to 4070 cycles/sec.). It is obvious that the deviation of this frequency from one of the 84 reference points cannot be greater than 50 cents. If a calibrated driving mechanism, adjustable to ±50 cents, be applied to the chromatic gear box (see Fig. 2) then all the 84 standards can be shifted by the same number of cents until one equals the unknown. The particular disk and pattern which are stroboscopically stationary indicate the nominal frequency (see the bracketed part of Eq. (4)) and the deviation from standard may be read directly from the single calibration, regardless of whether the unknown frequency be large or small.

The requirements of such a calibrated adjustable drive are rather severe. It must be capable of being adjusted to any speed within the ±50 cents range (less than ±3 percent), and of maintaining that speed well within 1 cent (better than 1/20 percent). In addition the calibration must be stable and provide 100 units over the adjustable range. This necessitates a synchronous motor fed from a power supply of more nearly constant frequency than is normally available from the commercial lines.

Figure 3 shows one method which has been used to provide such a drive. The synchronous motor is fed from a fork-controlled power supply, and runs at 30 r.p.s. This drives one side of a differential, the ring gear of which is rotated by a variable speed motor, either of the series type with friction governor, or synchronous with variable ratio drive mechanism. An electric tachometer, having a depressed zero meter, is arranged to read zero on its scale when the variable speed motor is set for 2250 r.p.m. With this adjustment the drive shaft to the chromatic gear box revolves at 27.5 r.p.s. If the variable speed motor is adjusted to give 1524.8 r.p.m. the drive shaft speed is increased by 50 cents, and the meter reading is +50 cents. Similarly any other deviation from standard is shown on the 100 division scale of the tachometer, which is marked to read cents directly.

This arrangement fulfills the requirements set forth above and has the advantage over that previously used[1] in that the differential ring gear is always in motion. Nevertheless fine machine work on the differential is necessitated. Since a tuning fork has already been provided to obtain the desired constancy, it has been found more satisfactory to incorporate the adjustable feature in the fork, and thus to eliminate the mechanical differential entirely. Such a calibrated adjustable tuning fork is described elsewhere in this journal.[6] Fig. 1 shows the fork unit at the right. The fork is, of course, designed to satisfy the requirements already noted: a frequency adjustment whereby the speed of the synchronous driving motor may be set to 27.5 r.p.s.±50 cents, and a 100 division calibration for this range. The chromatic gear box speeds thus remain as described above, and the 84 reference points are dependent upon the single tuning fork.

THE FLASHING AMPLIFIER

Figure 4 shows the circuit used to produce light flashes synchronized with the frequency of the sound incident upon the microphone. Since frequency doubling is not desired, a bias voltage is required on the gaseous discharge tube. This bias is conveniently provided by the plate voltage supply of the last tube, and the a.c. output of the amplifier superposed upon the d.c. bias by means of the auto-transformer.

Since the present device is primarily designed

[6] See following paper by O. Hugo Schuck entitled "An Adjustable Tuning Fork Frequency Standard."

for the measurement of frequency, the intensity of the light flash need not be related to the original sound intensity. Indeed for testing piano strings having a high decrement Dr. O. L. Railsback suggested the utility of making the light flash intensity independent of the sound amplitude.

The wiring diagram of Fig. 4 shows a method due to O. H. Schuck which tends to produce this effect. When the swing of the output tube's control grid becomes but slightly greater than is required to produce flashing on the negative peaks, the positive peaks begin to draw grid current. The average potential drop due to this grid current flowing through the grid resistor is applied through a resistance-capacitance filter to add to the negative bias of the second-stage amplifier tube. This being a variable-mu tube, its amplification is thus reduced in accordance with the overloading in the output stage. With this arrangement the input voltage to the amplifier may be varied over a range of 60 decibels and the stroboscopic image produced will still be satisfactory.

During the time that current flows in the gaseous discharge tube the light intensity is dependent upon the current wave form. Within certain limits this makes it possible to measure the frequencies of two unrelated sounds produced simultaneously, or the component frequencies of a complex tone, even if inharmonic.

The stroboscopic effect can of course be produced by either transmission or reflection of the light. Even though a roughened disk surface is used the latter method is hampered by spectral reflection. The transmission arrangement has the advantage of making more efficient use of the available light. Following experiments with perforated disks, a system was devised using translucent disks and a U-shaped discharge tube placed behind the disks in line with the openings in the mask. Bright patterns are thus visible over a large angle, and spectral reflection is eliminated.

Fundamentals

From the description given above it will be observed that aside from the null indicator the measuring portion of the chromatic stroboscope consists of three parts:

(1) A system of interconnected standards, or reference points, successively related to each other by a common basic ratio (the disk with its seven rings, the numbers of segments being related by the 2/1 ratio),

(2) a subdivision of the basic ratio into a given number of equal ratios, also interconnected (the chromatic gear box with its twelve speeds successively in the $2^{1/12}$ ratio),

(3) a calibrated adjustable portion so located as to change all the interconnected standards and subdivisions by the same ratio, and capable of being adjusted and read over a continuous range equal to the subdivision (the adjustable tuning fork changing the speed of a synchronous motor by ± 50 cents).

A precision in frequency determination of about 1/20 percent may be had by this arrangement. This is made possible by having the continuously adjustable calibrated element cover only a small part of the total range in which measurements are to be made. Such an artifice is of course common, but the location of the fine adjustment feature in the present device gives a new result.

Many of our precision measuring devices comprise a set of standards, their subdivisions suitably chosen, and a calibrated adjustable portion *that may be added thereto*. An example is the micrometer marked off in lengths of 1/10 inch, subdivided at each 1/40 inch, leaving the latter to be covered continuously by the calibrated head marked off in 25 divisions. Radio-frequencies are sometimes measured by adding a known audiofrequency from a calibrated oscillator to one of a series of accurately known harmonics from a crystal controlled oscillator thus obtaining a frequency which may be shown equal to the unknown by some suitable null indicator.

In apparatus of the type illustrated by these examples the adjustable element is always in effect added to the standard reference point, and the method may be termed *algebraic*. In the chromatic stroboscope the basic standards and subdivisions are made dependent upon the adjustable portion, but without changing their ratios. This method may be called *geometric* due to the fact that a standard multiplication of the variable element is involved so that one may read directly on the basis of ratios, or rather their logarithms.

Furthermore, the geometric method is not limited to the single set of subdivisions as used

in the chromatic stroboscope. For example, a change gear box providing ten speeds successively in the ratio of $2^{10/1200}$ could be interposed between the chromatic gear box and the adjustable drive. The latter then would need to cover but ± 5 cents, and a 100 division calibration would make it possible to read to 0.1 cent. The basic standards and interconnected subdivisions may be placed in any convenient order, but for a continuous reading device they must be made dependent upon the calibrated part which can be adjusted over a range represented by the smallest subdivision ratio.

This method of arranging basic ratios and constant ratio subdivisions so as to be dependent upon an adjustable standard is general in that it is applicable to the precise measurement over a continuous range of the ratio of any quantities, whether they be lengths, frequencies, or electrical voltages.

An Illustration of the General Approach

By using the fundamental scheme just outlined it is possible to design in a straightforward fashion other apparatus whereby logarithmic units may be read directly. Indeed since an electrical counterpart exists for each element of the chromatic stroboscope, its logarithmic measuring features could be embodied in a purely electrical device.

Suppose it is desired to measure frequencies for musical purposes in terms of the perfect 12th as the basic interval, i.e., the 3/1 ratio. For a stroboscopic arrangement such as described above, the disk might have four rings of 2, 6, 18 and 54 dark segments respectively. Then to fill in the 3/1 ratio with semitone intervals of about the usual size a chromatic gear box having 19 shafts rotating successively in the ratio $3^{1/19}$ is necessary. Actually this new semitone is slightly larger than the one defined by $2^{1/12}$ so that the 89/84 ratio used throughout provides the proper relationship for such a gear box. A calibrated drive, now adjustable over the range $3^{1/19}$ is needed to make the device capable of being read over a continuous range. This last feature of course is not required if comparison is to be made only at the standard reference points furnished by the disk and gear box.

Acknowledgment

We are indebted to Dr. O. L. Railsback of Charleston, Illinois, who brought us a scheme for driving a single stroboscopic disk by superposing a variable speed through differentials upon a series of shaft speeds derived from change gears. In attempting to adapt this algebraic arrangement to musical use we conceived and developed the ratio-reading method described above.

An Adjustable Tuning Fork Frequency Standard

O. Hugo Schuck
C. G. Conn, Ltd., Elkhart, Indiana

A portable tertiary frequency standard, adjustable over a small range, is described. The frequency determining element is a tube-maintained tuning fork, adjustment of whose frequency over a relatively small range is accomplished by changing the position of movable weights on the prongs. Calculations are simplified by the design of the fork, which permits the assumption of lumped constants.

The weights may be moved while the fork is running without interfering with its operation. Indication of their position is by means of a pointer reading on a scale graduated directly in frequency units. A precise method of calibration is given. Mechanical arrangements are described for obtaining a uniformly spaced scale, and the possibility of using such a scale in correcting for the temperature coefficient of the fork material is discussed.

The relative accuracy of such a fork is greater, the smaller the range of frequency adjustment. In the present development, adjustment may be made over a range of plus and minus three percent of the mean frequency, that is, a musical interval of one semitone, or 100 cents total change. Incorporation of this fork in the chromatic stroboscope has resulted in considerable simplification as well as an increase in the reliability of the instrument.

Introduction

TUNING forks are among our earliest and most widely used frequency standards. Although at first they were excited by striking or bowing, more recent practice is to maintain a fork in continuous vibration by electrical means. This allows more accurate comparison of the frequency being measured with that of the fork.

Adjustable tuning forks, having movable weights on the prongs[1] or similar arrangements[2] for adjustment of frequency, have also been used previously. It is the purpose of this paper to describe such a fork which has been developed to serve as a portable tertiary standard for frequency measuring purposes.

General Description

Figure 1 shows a picture of the tuning fork being described, as assembled with movable weights and scale mechanism. Adjustment during operation is accomplished by rotating the knob *A* on the front of the assembly, thus moving the carriage *B* through the tapes and pulleys, and thus the weights *C* which are connected by flat push springs to the carriage. The pointer, which is free to rotate about the knob shaft, is coupled to the carriage by a link, and indicates on the scale the deviation from the reference frequency.

Figure 2 is a drawing of the tuning fork alone, the design of which is due to Allen Loomis and R. W. Young. This design was chosen to allow economy in the use of material and space, and to avoid a concentration of bending stress while still inhibiting any mode of vibration but the fundamental mode. It is relatively easy to machine accurately,[3] and calculations made with the assumption of lumped constants may be used in determining both its dimensions and the effect of the movable weights.

While an expression is available for the frequency of a plain fork,[4] its application depends upon the fork prongs being beams of constant thickness. A weight placed on such a fork prong would alter its bending curve, and would therefore change the effective compliance, in addition to having its expected effect in increasing the moment of inertia. The center of rotation about

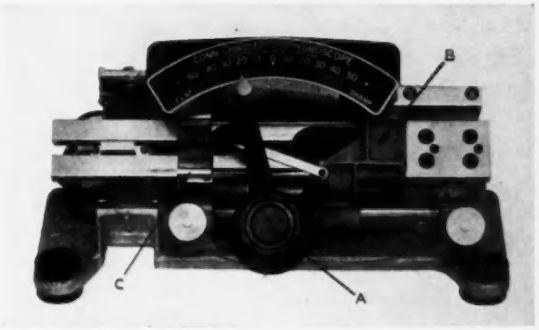


Fig. 1. Adjustable tuning fork.

[1] Rayleigh, *Theory of Sound* (Macmillan, 1929), Vol 1. p. 59.
[2] K. Lehman, Zeits. f. tech. Physik **18**, 304 (1937).
[3] H. W. Lamson, J. Acous. Soc. Am. **7**, 51 (1935).
[4] A. B. Wood, *Textbook of Sound* (Macmillan, 1930), p. 119.

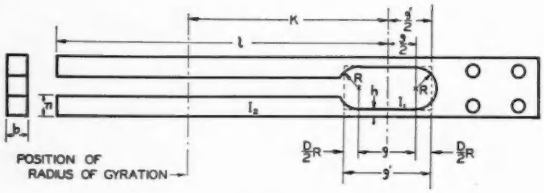


FIG. 2. Tuning fork alone.

which the moment of inertia of the weight is calculated would furthermore vary with the position of the weight.

In this fork design, the functions of furnishing compliance and inertance are physically nearly independent. The thin portion near the heel acts as a flat spring, and practically no bending takes place in the main portion of the prong, which is many times as stiff as the bending portion. It is found experimentally that a fork of this design vibrates substantially as though the heavy portion of the prong were pivoted at a point at the center of the bending portion, and that the effective compliance of the bending portion remains constant when extra weights are added to the prongs.

By using a section of uniform thickness for the bending portion, a concentration of stress at any one point is avoided. The low maximum stress thus obtained makes changes due to fatigue unlikely to occur.

FORK CALCULATIONS

Calculations may be made with the assumption of lumped constants, that is, that the bending curve of the compliant portion is always the same, and that the prong swings as though pivoted about a fixed point. The results will not be exact because of the factors mentioned above, as well as the effect of the push springs that control the position of the movable weights. However, these differences are relatively small, so that a fork may be built coming fairly close to the calculated values.

Let us consider the tuning fork shown in Fig. 2, with the dimensions there given, and treat one prong as a vibrating system pivoted at the center of the bending portion. A uniform bar will have a moment of inertia about this point of

$$I_0=\rho bnl^3/3 \tag{1}$$

if the moment of inertia of the small cut-out be neglected. The mass is

$$M_0=\rho bnl \tag{2}$$

and the radius of gyration is

$$K=(I_0/M_0)^{\frac{1}{2}}=l/3^{\frac{1}{2}}. \tag{3}$$

Considering now that the mass of the bar is concentrated at the radius of gyration distant K cm from the center of rotation, the linear compliance acting at this point may be taken as that of a cantilever beam of two sections, having respective section moments of inertia I_1 and I_2 as indicated in Fig. 2 and modulus of elasticity E. As developed in the appendix, the compliance of such a beam to a force acting at the radius of gyration is

$$C_K=\frac{1}{EI_2}\left[\frac{(K+g'/2)^3}{3}+g'\left(\frac{I_2}{I_1}-1\right)\left(K^2+\frac{g'^2}{12}\right)\right], \tag{4}$$

where

$$g'=g+DR. \tag{5}$$

To obtain the effective bending length g', it is necessary to increase the distance between centers g to take account of the bending in the portions bounded by the circular arcs. The value of the factor D depends upon the ratio of the thickness h to the radius R, and is given by

$$D=-0.543+1.875(h/R)^{\frac{1}{2}}. \tag{6}$$

The derivation of this expression is described in the appendix. For the particular fork described later in this paper, the value of D is close to 0.90.

From the linear compliance C_K the rotational compliance C_r is obtained by

$$C_r=C_K/K^2, \tag{7}$$

and the frequency of the fork without weights is

$$f=1/2\pi(C_rI_0)^{\frac{1}{2}}. \tag{8}$$

In Fig. 3 is shown a fork with movable weights of mass m (this includes one-third of the mass of the push springs), whose centers of gravity are at a distance x from the center of rotation of the prong. Its frequency f_x will be a function of the position of the movable weights. The

moment of inertia of one prong with its weight is

$$I = I_0 + I_m + mx^2. \tag{9}$$

I_m, the moment of inertia of the weight about its own center of gravity, may be lumped with I_0, that of the prong about its center of rotation, to give

$$I' = I_0 + I_m, \tag{10}$$

and the frequency expression (8) becomes

$$f_x = \frac{1}{2\pi(C_r)^{\frac{1}{2}}} \cdot \frac{1}{(I' + mx^2)^{\frac{1}{2}}}. \tag{11}$$

It is obvious from this equation that f_x is not a linear function of x. An experimental curve showing the frequency variation produced by moving the weights is given in Fig. 4. It was convenient in this case to plot frequency ratios, expressed in cents deviation (1 cent equals 0.06 percent) from 55 cycles, instead of actual frequencies, although a frequency scale is also given. These deviations are plotted against distances measured toward the heel of the fork from a zero position which is chosen arbitrarily to give an equal range of cents deviation above and below the required mean frequency of 55 cycles.

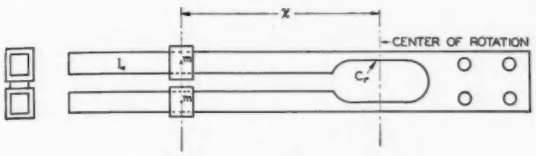


FIG. 3. Tuning fork with weights.

SCALE RECTIFICATION

From the curve in Fig. 4 a scale could be constructed, such as is given in the upper part of the figure, against which the position of the weights could be read to obtain the deviation. It is seen that the divisions are much more crowded at the *minus* end than at the *plus* end. While satisfactory for laboratory work, the nonuniform appearance of this scale would be confusing and misleading to nontechnical users, such as musicians, whose requirements have been carefully considered in this development. Mechanical means are therefore required for rectifying the scale so that a pointer may read the deviations on a uniformly graduated scale. Various mechanical arrangements were considered and the one adopted is illustrated diagrammatically in Fig. 5.

It is seen in the end view that the weights consist of U-shaped pieces fitting the thickness dimension of the prong with a sliding fit and held snugly in the direction of vibrational motion by clamping springs. These allow easy longitudinal adjustment along the prong, but prevent any relative transverse motion, that is, in the direction of vibration.

Longitudinal position is controlled by the push springs which are attached to a carriage moving in guides, the guides being omitted from the diagram. An arm attached to the carriage transmits its motion, through the link, to the pointer which rotates about a stationary pivot. The length of the link and the position of its pivotal attachment to the pointer and carriage are so chosen that the necessary variable ratio is introduced in translating the linear motion of the carriage into angular motion of the pointer. The result of using this arrangement is shown in Fig. 6, in which is given a representative measured curve of angular motion of the pointer against cents deviation in the frequency of the fork. It is seen that only at the ends of the range is there any appreciable departure from linearity, and that these departures produce a symmetrical effect in that they slightly stretch the graduations at both ends. A scale thus graduated is seen in Fig. 1. Since the pointer may be of any length, any desired scale length can be obtained.

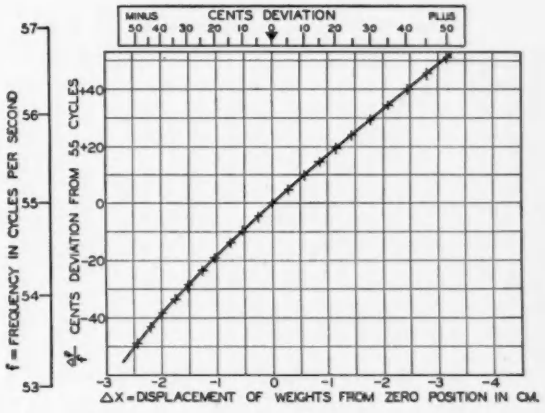


FIG. 4. Frequency deviation *vs.* position of weights.

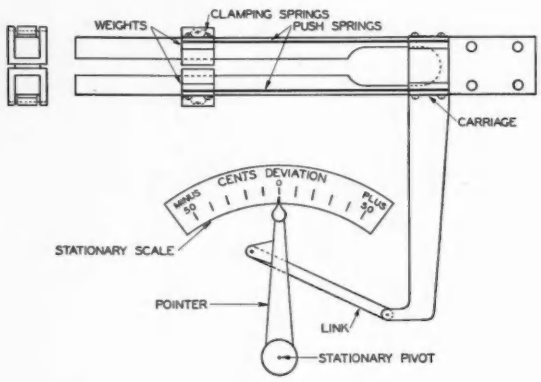


FIG. 5. Scale rectification by link motion.

TEMPERATURE CORRECTION BY MOTION OF SCALE

In addition to the improvement in appearance, a uniform scale offers a means of correcting for the effect of temperature upon the frequency of the fork. As is well known, the modulus of elasticity of most metals decreases with increase of temperature. Tuning forks made of such metals will accordingly decrease in frequency as the temperature increases, as shown in Fig. 7. Over the ordinary range of ambient temperatures, this variation in frequency is directly proportional to the temperature, and for various materials has the values shown. For a cold-rolled steel fork, for example, the frequency changes by 1 cent for less than 5°C (9°F). By moving the scale itself by the proper amount, either manually or automatically by means of a bimetallic strip, the same compensation is introduced in all parts of the uniform scale. There is, to be sure, a slight error at the ends of the scale because of the departure from linearity, but it is of minor importance. A method is thus available for building an accurate adjustable tertiary frequency standard whose readings are not affected by temperature, by using for the tuning fork ordinary, easily obtained cold-rolled steel, for example.

While the arrangement for temperature correction described above was actually used in some of our development models in which the fork was made of cold-rolled steel, it was decided in setting up the production design to eliminate the difficulty at its source. A special 36 percent nickel–12 percent chromium alloy steel is therefore used for the fork at present. Its temperature coefficient of frequency is about −10 parts per million per °C, so that within a reasonable temperature range no correction is necessary for ordinary work.

DESIGN PRECAUTIONS

While the adjustable tuning fork only has been described above, it is used with an amplifier for maintaining its vibration and for furnishing electrical power of the fork frequency to a synchronous motor. A wiring diagram of this amplifier is shown in Fig. 8. It is seen that the first two stages, which are arranged in a regenerative amplifier circuit with the fork as the feed-back element, serve only to drive the fork, the input to the driver stage of the output amplifier being in parallel with the grid of the second stage. The first stage acts solely as a voltage amplifier, the amplitude-limiting function being performed by grid overloading in the second stage.

While the first stage could have been eliminated, as is often done,[5, 6] its use allows the pick-up and drive magnets, which are telephone receiver assemblies, to be placed farther from the fork, thus reducing the eddy current damping. That this is an important point is illustrated by the data plotted in Fig. 9. To obtain these data an extra drive-magnet assembly was moved with respect to the fork and the resulting frequency change noted. The fact that small

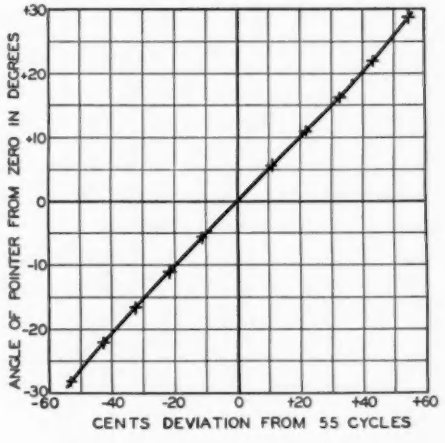


FIG. 6. Scale calibration curve.

[5] S. Butterworth, Proc. Phys. Soc. **32**, 345 (1920).
[6] E. Norrmann, Proc. I. R. E. **20**, 1715 (1932).

changes in the air gap have less effect on the frequency when the air gap is large than when it is small is readily apparent. Such small variations can readily occur if the fork base is not level, as it was when calibrated, so that the weight of the prongs causes them to deflect from their calibration position. They can also occur as a result of permanent deformations of the fork caused by shocks in transporting it. In this respect the low maximum stress permitted by the relatively long bending portion is a distinct advantage, as the pole pieces act as stops to limit the possible motion to a value such that at no time can any fiber be stressed near its yield point. To provide such stops to motion in both directions and to reduce the effect of mis-leveling, as well as to insure that the damping is balanced, a symmetrical drive and pick-up magnet system is used. As seen in Fig. 1, the pole pieces of the pick-up coil extend up between the prongs of the fork, and the drive coils act on the outside of each prong. The effectiveness of this arrangement with respect to jars in transportation is attested to by the fact that from hundreds of miles of travel in an automobile trunk and by train shipment, no frequency variation of as much as half a cent

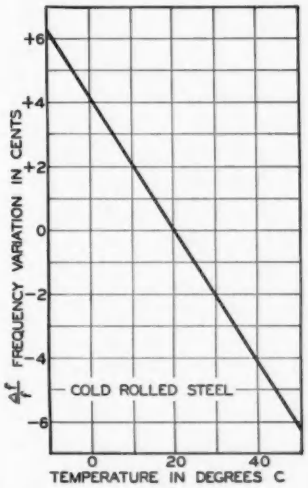


FIG. 7. Frequency deviation *vs.* temperature.

MATERIAL	FREQUENCY VARIATION PER DEGREE C PARTS PER MILLION	CENTS
Aluminum 17ST	−285	−0.48
Stainless 18–8	−220	−0.37
Ketos Hardened	−140	−0.23
Cold Rolled Steel	−125	−0.21
Spring Steel	−105	−0.18
36Ni–12Cr Steel	− 10	−0.017

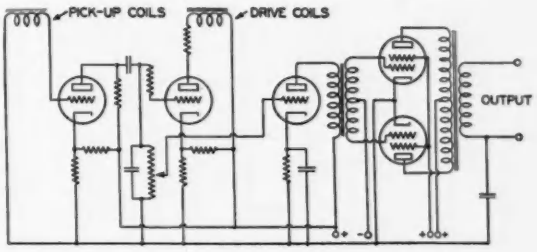


FIG. 8. Fork amplifier circuit.

resulted. With respect to mis-leveling, the curve of Fig. 10 shows that the fork assembly may be considerably tilted without causing serious error. Contributing to this stability is the rigid angle-section construction of the cast aluminum base upon which the fork and the magnet assemblies are mounted, and which is supported at three points on rubber bushings.

Inclusion of the voltage amplifier stage also allows the effect of line voltage variations to be minimized without using an expensive voltage regulator tube. The wiring diagram (Fig. 8) shows that the cathode resistors are not by-passed, and that additional current from the bleeder resistors flows through them. A form of inverse feed-back is thus achieved which, with the automatic variation in grid bias obtained, results in reasonably constant amplification despite changes in supply voltage. Fig. 11 shows the effect on the frequency due to variation of the line voltage, the voltage coefficient being -0.0045 cent per volt (-2.7×10^{-6} per volt) at a line voltage of 115 volts.

CALIBRATION OF SCALE

It remains to describe the method of accurately calibrating the scale on which the frequency adjustment is read. The adjustable tuning fork frequency standard here treated was developed for use in the chromatic stroboscope, which is described elsewhere.[7] In this application the reference frequency is 55 cycles per second, and the range of adjustment is from 50 cents below to 50 cents above this reference frequency. Graduations must be provided at one cent intervals within the range of -50 cents to $+50$ cents, and the same degree of accuracy must be attained throughout this range. In the reference[7]

[7] A companion paper by R. W. Young and A. Loomis entitled "Theory of the Chromatic Stroboscope."

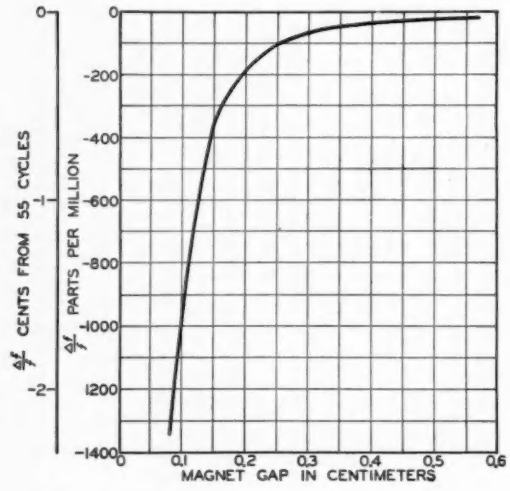


FIG. 9. Frequency deviation *vs.* magnet gap.

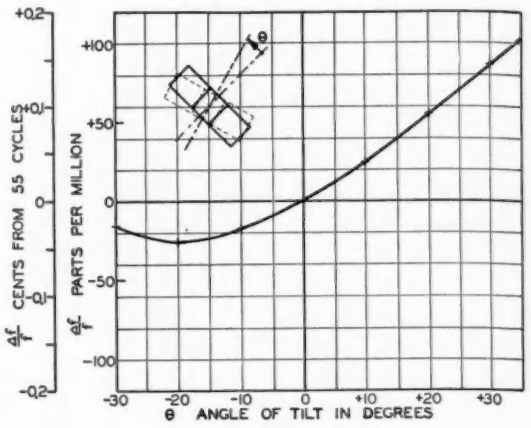


FIG. 10. Frequency deviation *vs.* angle of tilt.

it is explained that a disk having sixteen sets of alternate light and dark segments will show a stationary pattern under 440 cycle per second intermittent illumination if it is driven at 27.5 revolutions per second. This speed will be obtained with a four-pole synchronous motor operating on current of frequency 55 cycles per second, such as is supplied by the output of the fork amplifier described above. With the pointer at zero on the scale a stationary pattern will thus result if the adjustable tuning fork is then vibrating at exactly 55 cycles per second. Adjustment of the fork to do so is made by selection of the weights of standard set-screws inserted into tapped holes in the ends of the prongs, and prevented from loosening by being drawn up firmly against the bottoms of the holes. Final adjustment is accomplished by slightly moving the scale itself.

For the reference frequency of 440 cycles per second, mentioned above, the standard frequency emissions of the National Bureau of Standards may be used under favorable conditions. However, these come to Elkhart, Indiana, by a transmission path involving ionosphere reflection and a considerable amount of fading is usually experienced. It is therefore more convenient to use a local secondary standard. Such a standard is available in the form of a temperature controlled, electrically maintained tuning fork, also operating at 440 cycles per second, and frequently checked against the 440 cycle per second standard frequency emissions. Its frequency is known to within 5 parts per million or 0.008 cent.

In order to graduate the scale from -50 cents to $+50$ cents it is necessary to have a number of calibration points which are all known to the same degree of accuracy as the one at the zero position. A frequency multiplier operating on the output of the 440 cycle per second tuning fork secondary standard gives an output of 4400 cycles per second with an accuracy the same as that of the fork. If the 16-segment disk referred to above is replaced by one having 160 segments and illuminated by light flashing in accordance with the 4400 cycle per second output of the frequency multiplier, a stationary pattern will again result if the tuning fork driving the synchronous motor is vibrating at 55 cycles per second. With the same illumination frequency a disk having 159 segments will likewise show a stationary pattern if the tuning fork frequency is 55.346 cycles per second ($=55\times160/159$), or $+10.85$ cents on the scale. Similarly a 158 segment disk will show a stationary pattern at $+21.78$ cents, and so on.

A single large disk is actually used in performing the calibration. It is illustrated in Fig. 12. As indicated in the figure, it has eleven rings of holes, the inner ring having 155 holes, the next 156, and so on to 165 holes in the outer ring. For each ring there is given the actual fork frequency for stand-still and the corresponding interval in cents from 55 cycles. Eleven derived reference points are thus obtained, slightly more than covering the required range,

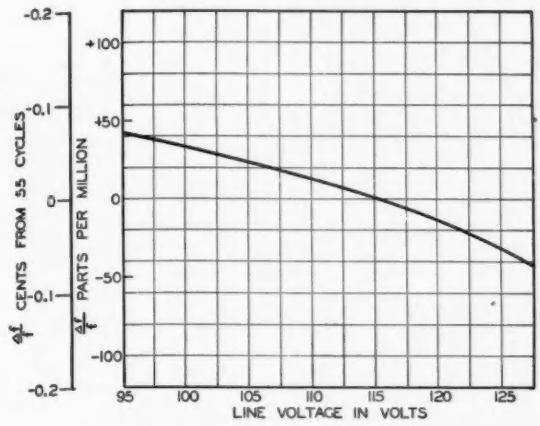


FIG. 11. Frequency deviation *vs.* line voltage.

and each having the same accuracy as that of the 440 cycle per second standard.

The frequency of the adjustable fork being calibrated is adjusted by moving the weights so that each of the eleven rings of holes stands still in turn, and the corresponding pointer angles are noted. It should be mentioned that the accuracy of this adjustment is considerably enhanced by the frequency multiplication and by the increased size of the disk, so that any drift of the pattern is correspondingly more quickly apparent. The pointer angles obtained at each of the calibration points are plotted against their deviations in cents as given in Fig. 12 and a curve is drawn. Such a curve is given in Fig. 6. By reading from the curve, the angular positions of the pointer for the required graduations are determined, and they are engraved on the metal scale. At present this work is being done by hand, but an engraving machine is being built which, in addition to doing the engraving, will also perform the necessary interpolation.

It has been found necessary, even though the most careful control is exercised in the manufacture of the tuning forks and the movable weights, to graduate the scale for each fork assembly individually in order to attain the desired accuracy throughout the required range of frequency variation.

ACCURACY

Experience over a period of several months with five of the adjustable forks here described and their associated amplifiers, added to a consideration of the data given above on the magnitude of temperature and voltage effects, leads to the conclusion that frequency measurements made using as a standard the adjustable fork here described may be relied upon to better than 1 cent (0.06 percent). This accuracy is attained for measurements throughout the continuous range of 32 to 4070 cycles per second.

ACKNOWLEDGMENTS

Mr. Allen Loomis and Dr. Robert W. Young, both colleagues of the writer in the Experimental Department of C. G. Conn, Ltd., contributed greatly in ideas and suggestions to the developments here described. The Engineering Department of the same organization is to be credited with much of the mechanical design for the production model.

APPENDIX

Compliance of two-section cantilever beam

One prong of the tuning fork shown in Fig. 2 is to be considered as a cantilever beam, with the origin of coordinates taken at the right hand end of the effective length g' of the bending portion. Suppose a force of P dynes is applied at $x=(K+g'/2)$ cm, K being the radius of gyration of the prong about a center of rotation assumed at the middle of the bending portion. A deflection of y_K cm is produced. The effective linear compliance at this point is

$$C_K=(y_K/P)\text{cm/dyne}. \tag{12}$$

The moment at any point is

$$M=-P[(K+g'/2)-x]\text{ dyne cm}, \tag{13}$$

from which the differential equation of the elastic curve is[8]

$$EId^2y/dx^2=-P[(K+g'/2)-x]\text{ dyne cm}, \tag{14}$$

where E is the modulus of elasticity of the material in dynes per cm^2 and I is the section moment of inertia in cm^4.

The cantilever beam consists of two parts, one of length g' and section moment of inertia I_1, one end of which is fixed, and the other of

[8] J. E. Boyd, *Strength of Materials* (McGraw-Hill, 1924), p. 145.

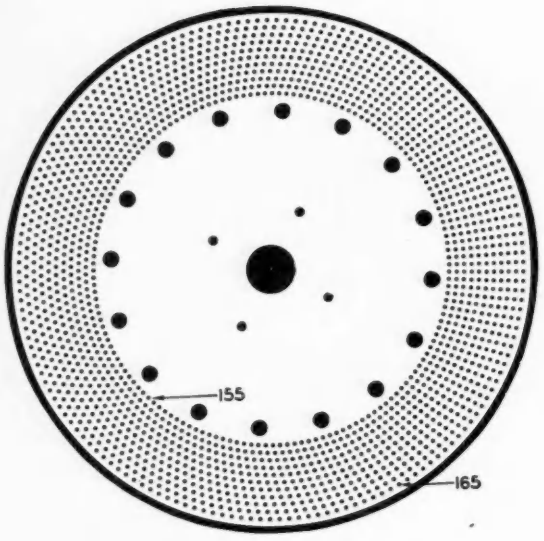


FIG. 12. Disk for calibrating scale.

NUMBER OF HOLES	FREQUENCY FOR STAND-STILL	CENTS FROM 55 CYCLES
155	56.774	+54.96
156	56.410	+43.83
157	56.051	+32.76
158	55.696	+21.76
159	55.346	+10.85
160	55.000	0.00
161	54.658	−10.79
162	54.321	−21.50
163	53.988	−32.16
164	53.659	−42.74
165	53.333	−53.27

length $(K-g'/2)$ and section moment of inertia I_2. One equation of the form of (14) must be set up for each part and the two solved simultaneously. For the thin portion, the boundary conditions are that when $x=0$, $dy/dx=0$ and $y=0$. For the thick portion, the boundary conditions are that the slope and deflection of both parts shall be respectively the same at their junction where $x=g'$. The deflection at the point of application of the force P is

$$y_K=-\frac{P}{EI_2}\left[\frac{(K+g'/2)^3}{3}+g'\left(\frac{I_2}{I_1}-1\right)\left(K^2+\frac{g'^2}{12}\right)\right], \quad (15)$$

and the effective linear compliance is

$$C_K=\frac{1}{EI_2}\left[\frac{(K+g'/2)^3}{3}+g'\left(\frac{I_2}{I_1}-1\right)\left(K^2+\frac{g'^2}{12}\right)\right]. \quad (4)$$

Effective length of bending portion of fork prong

Referring to Fig. 2, it is seen that the bending portion of the tuning fork prong has a thickness h over the length g and that this increases at each end as the inner straight line joins the circular arcs. It is required to find the length g' of a bending portion of constant thickness h throughout which will give the same compliance as does the actual form. If the stiffness of the thick portion of the prong be considered very large, Eq. (4) in the previous section becomes

$$C_K=\frac{g'}{EI_1}\left(K^2+\frac{g'^2}{12}\right), \quad (16)$$

and if the bending portion is reasonably short compared with the lever arm K at which the force P acts $(K^2\gg g'^2/12)$, this reduces to

$$C_K=K^2g'/EI_1. \quad (17)$$

The compliance then depends directly upon the effective length of the bending portion, and in the case in which I_1 varies with position,

$$C_K=\frac{K^2}{E}\int_{v=a}^{v=a+g'}\frac{dv}{I_1}. \quad (18)$$

For the conditions here considered, the integral may be split into three parts, one for the constant thickness center portion of length g, and two equal parts for the two end portions bounded by the circular arcs. If an origin of coordinates be taken at the point where the center portion joins the right hand portion bounded by the circular arc of radius R, the thickness h_x at a point distant x from the origin is

$$h_x=h+y=h+R-(R^2-x^2)^{\frac{1}{2}}. \quad (19)$$

The section moment of inertia at this position is

$$I_x=\frac{bh_x^3}{12}=\frac{b}{12}(h+R-(R^2-x^2)^{\frac{1}{2}})^3, \quad (20)$$

and (8) becomes

$$C_K=\frac{K^2}{E}\left[\frac{g}{I_1}+2\cdot\frac{12}{b}\int_{x=0}^{x=R}\frac{dx}{(h+R-(R^2-x^2)^{\frac{1}{2}})^3}\right]. \quad (21)$$

If the second term in the bracket is replaced by

DR/I_1, the expression becomes

$$C_K=(K^2/EI_1)(g+DR), \tag{22}$$

which is the same as (17) if

$$g'=g+DR. \tag{23}$$

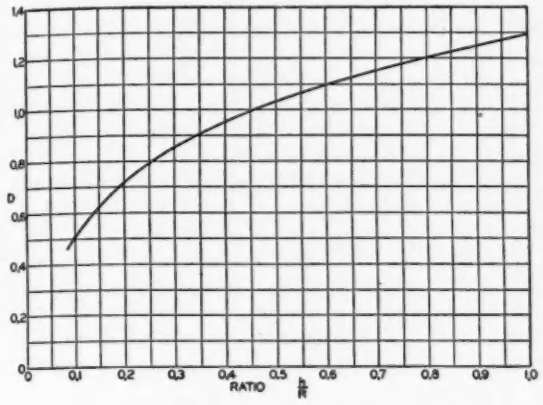


FIG. 13. Factor D *vs.* ratio h/R.

The length of the straight portion g must thus be increased by a factor D times the radius R of the arcs bounding the end portions in order to obtain the effective length g' required in (4).

The evaluation of the integral in (20) has been carried out for a number of values of h and R, and the resulting value of D as a function of the ratio h/R is plotted in Fig. 13. The curve fits closely the equation

$$D=-0.543+1.875(h/R)^{\frac{1}{2}}. \tag{6}$$

Implicit in the above derivation is the assumption that the beam is at all points symmetrical about the neutral plane, that the rate of change of the section moment of inertia is small, and that simple beam theory is thus applicable. Although this is certainly not strictly true for the case here considered, the values obtained give results close enough to measured values to be useful for design purposes.

Recent Advances in the Use of Acoustic Instruments for Routine Production Testing

BLAIR FOULDS
Electrical Research Products Inc., New York City
(Received June 9, 1938)

AS a result of the work described, we find that acoustic instruments can be used in a practical way for routine testing of products in which noise control is in itself a requirement, or of products in which some acoustic quality can be used as a measurement of precision or quality. In the former group are such products as electric motors, bearings, etc., in which it is desired to limit noise and, on the other hand, auto horns and signaling devices in which a limit is set on minimum output.

The data shown in the accompanying figures have been taken from the files of the Technical Consulting Department of Electrical Research Products, covering projects of this type.

Early attempts to use noise meters for this purpose were generally unsuccessful and as we review those cases we believe that most of the failures can be attributed to two principal causes, which we may for the moment call lack of frequency discrimination and lack of sufficient stability in the instrument.

A typical experience was somewhat like this: Noise level readings were made of several motors which were agreed to be good, and the readings noted. A noisy motor was then selected and tested in the same position relative to the microphone. Frequently the reading fell in the same range as that of the good motors. Seldom did it fall far enough above to be considered an acceptable test method.

Generally as a result the whole idea was condemned with the comment that "these noise meters don't hear the noise the same way as the ear," and as we learned from subsequent experience, the statement was usually correct. The noise meter by definition was intended to measure the level or, broadly speaking, loudness of a sound. The observer, on the other hand, in using his ear not only judges loudness of the sound as a whole, but involuntarily analyzes the sound, fixes his attention on any components which stand out or which he believes to be abnormal in the product, and judges the loudness of them individually.

As we see it, the inspector using aural methods listens to a tone in a motor, which to him denotes a defect, by comparing its intensity to that of another tone in the same motor. Furthermore, he makes this comparison in the presence of one or more other tones which may vary in masking value. And to make the situation still worse, he is probably working in the open plant or in a partially sound-proofed room in the presence of varying degrees of plant noise. It is not a shift in the threshold of his ears, or the grouch he has the morning after the night before, that causes him to reject motors which he might have passed some other time, but the variable conditions under which he has to listen. It was felt then that if a stable electrical system to do the listening could be worked out, the solution would be much nearer.

In Fig. 1 are shown the frequency response curves of three identical 1/5 hp induction motors. They all show about the same general shape but differ materially in the height of individual components. The peak at about 4000 cycles in one motor may indicate a defective bearing which the inspector detects as a distinctive tone. He listens to the tone and tries to evaluate it by comparing it with the hum components in the range below 300 c.p.s. If the hum components differ, as they do in the case illus-

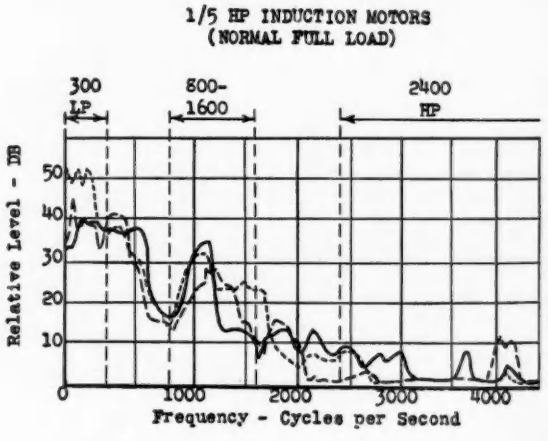
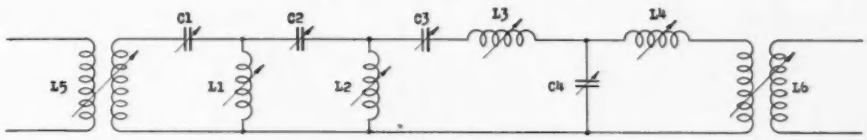


FIG. 1.



BASIC CIRCUIT DIAGRAM

Low-Pass Cut-offs C.P.S.		High-Pass Cut-offs C.P.S.		Band-Pass Filters C.P.S.	
50		50	75	50– 100	75– 150
100	150	100	150	100– 200	150– 300
200	300	200	300	200– 400	300– 600
400	600	400	600	400– 800	600–1200
800	1200	800	1200	800–1600	1200–2400
1600	2400	1600	2400	1600–3200	2400–4800
3200	4800		4800		

FIG. 2. Band-pass filters for routine industrial testing.

trated, the comparison is made on a different basis in each motor. In addition, the varying components in the 1000–2000 c.p.s. range may mask the components in question and further invalidate the comparison.

We had seen the inspectors of electric motors listen to their products and reject one which, on casual observation, seemed to us to sound just like all the rest. On listening more carefully, we had been able to detect the tone which meant to the inspector a tight bearing or a poorly seated brush, or an armature misalignment or some other defect. The solution seemed obvious; we would design filters which passed only the frequencies of these hums and whistles which meant defects. But to design these filters, we would have to first make frequency analyses of acceptable motors and those having various specific defects, in order to determine quantitatively what frequencies the defects introduced.

A number of these analyses were made and it was at this point that a significant factor came out, that is, that the presence of a defect generally introduced not a definite peak but a general increase in a portion of the spectrum, sometimes several octaves wide. This appears reasonable when we consider that many such defects produce a click or sound of shock nature having relatively steep wave fronts.

Figure 1 is typical of the frequency analysis of electric motors and motor-driven products and illustrates two significant points:

(1) That the over-all loudness of the motor is controlled largely by the low frequency components and does not reflect small changes in mid or high frequency ranges.

(2) That defects produce small changes in a portion of the frequency range, not pronounced peaks at definite frequencies.

At this step, the logical solution appeared to be in the use of a sound level meter, together with one or more wide band-pass filters which would pass the portions of the spectrum affected by various faults in the products under test. This, however, required frequency analysis of each different product to be tested, and design and construction of filters to fit each case. This was obviously not a flexible or convenient arrangement for a firm manufacturing several types of ball bearings or of motors, or of some other product. Commercial considerations, therefore, retarded development and it became increasingly evident that some more flexible system must be developed.

Some kind of a continuously variable wide band filter system similar to the heterodyne type frequency analyzers seemed to be about what we wanted from a technical standpoint, but we didn't see any way to get this in a simple rugged equipment suitable for routine plant use. In addition, we felt that a filter system having definite steps, while perhaps theoretically less flexible, would be much more practical and less likely to be operated incorrectly by the inspector.

The design of a filter system having a large number of steps was, therefore, attempted. A novel mechanical design was worked out utilizing a limited number of coil and condenser groups which are tied together in various combinations by a rather elaborate tap switch. Cut-offs at one-half octave intervals are provided and arrangement is made for selecting high-pass,

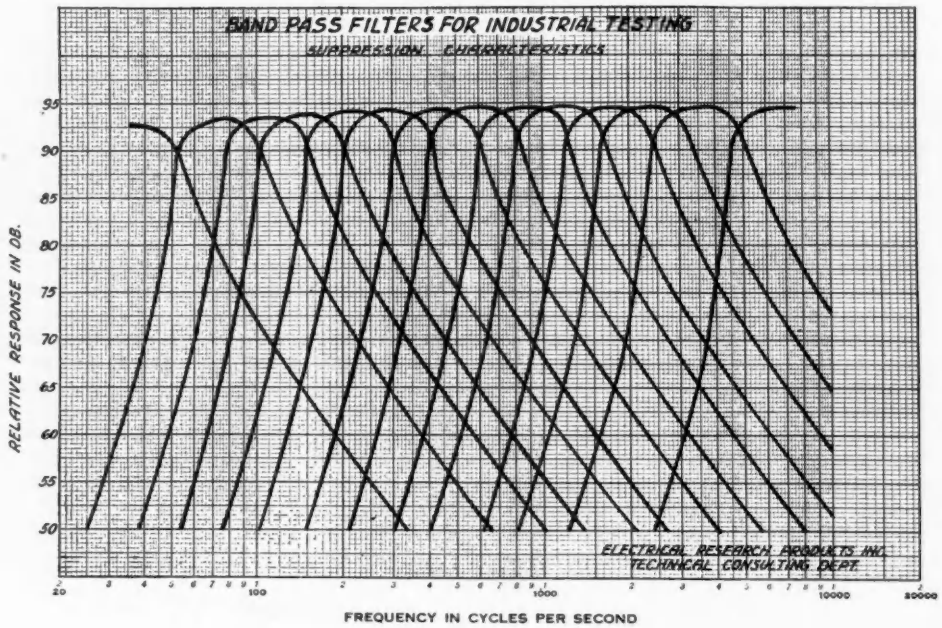


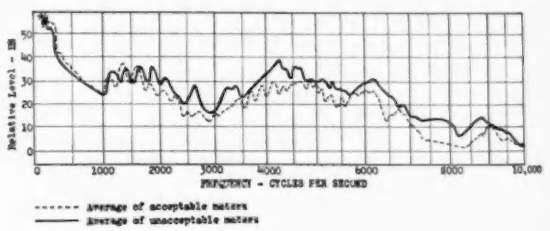
FIG. 3.

low-pass or octave-width band-pass characteristics. Experience to date has indicated that the arrangement is sufficiently flexible to meet the requirements of most products. The basic circuit is shown in Fig. 2.

As to suppression characteristics, it will be noted that, as pointed out previously in most products, the extreme low frequencies predominate with the result that the high-pass side of the filters must have a relatively high suppression. Constant "*k*" type sections appeared to be the best solution from the standpoint of simplicity; also the characteristic of continuously increasing suppression without repeat points was desirable. Double sections were used on the high-pass side and single sections on the low-pass side, providing a suppression of approximately 40 db and 20 db respectively at one octave away from the cut-off frequency. Suppression characteristics are shown in Fig. 3.

Completion of the filters enabled us to obtain much more actual data and experience than previously on a variety of products. Fig. 4 shows averages of frequency analyses of some electric fan motors. The over-all noise readings of the acceptable motors actually averaged higher than the unacceptable which completely eliminated the possibility of using over-all readings as a test method. A good spread in the high frequency range permits use of the 4800 cycle high-pass filter for satisfactory inspection.

Figure 5 shows typical curves on some small ball bearings. The dotted line is the average frequency response of bearings judged to be satisfactory and the solid line bearings having one type of defect. It will be noted that the two curves fall together over most of the range except from about 3500 to 6500 cycles. The readings taken in several octave bands are also indicated, and the spread in readings obtained in the 4800 c.p.s.



Filter Selection	Average Readings	
	Acceptable Motors	Unacceptable Motors
Over-all	62.8	62.5
1200 Low-Pass	60.4	59.6
1200–2400	51.1	52.5
2400–4800	50.9	52.3
2400 High-Pass	55.5	57.0
4800 High-Pass	48.6	52.3

FIG. 4. Frequency analysis of 110-volt series electric fan motors (loaded by dynamometer).

high-pass is obviously adequate for inspection purposes.

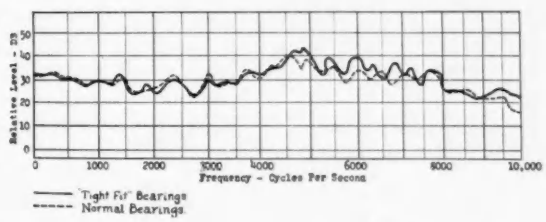
Another example is shown in Fig. 6 which illustrates data on small 6-volt fan motors for automotive use. In this case, the use of the 2400–4800 c.p.s. band is indicated.

At this point the basic principle of the system seemed to have been proved correct, and it remained to tackle some of the details necessary to make it practical for routine plant use..

The necessity for extreme stability of calibration was obvious. It was evident that acceptable and unacceptable products would not fall into definite groups but would be distributed continuously on both sides of a limiting value. If the calibration of the test instrument were to shift, it would be equivalent to shifting the acceptance value and the method would be no better than the use of the ear.

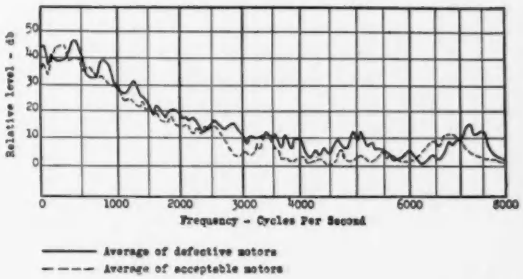
For plant use, a.c. operation was preferred. Previously a.c. operation of sensitive sound level meters had not been considered practicable because of the effect of line voltage and frequency variation on calibration, and the difficulty of overcoming plate supply ripple with high gain settings.

The use of an a.c. operated feedback amplifier was the solution to both of these problems. A feedback of about 35 to 40 db was used with a net gain of 120 db. For line voltage variations of 10 percent there is a grain variation of less than 0.1 db which we believe can be considered negligible for practical purposes. Vacuum tubes can be replaced without affecting calibration.



Filter Selection	Average Readings: Normal	Tight Fit
Over-all	69.5	70.5
75– 150	37.0	37.0
150– 300	37.3	37.5
300– 600	45.7	46.2
600–1200	51.0	52.5
1200–2400	53.5	54.2
2400–4800	66.3	66.0
4800–HP	67.1	72.1

Fig. 5. Average frequency response. Normal and tight-fit ball bearings. (1″ O.D.—radial load.)



Filter Selection	Defective	Acceptable
Over-all	58.5	58.5
2400 lp	57.1	57.9
2400–4800	46.0	41.3
4800 hp	44.8	44.8

Fig. 6. Six-volt auto heater motors, dynamometer loaded.

All the noise meters on which earlier experience had been gained had been designed to have ballistic characteristics per the A.S.A. standards and a rapid fluctuation was frequently obtained in readings. Various speeds of indicating element were tried and experience to date indicates that a speed of about 2 seconds for full scale deflection is generally best.

It will be noted in the above discussion that in all cases a frequency analysis was first made and the choice of filters made on the basis of these data. We have found it practicable, however, to work directly with a multi-step filter set of the type described without using a continuously variable analyzer. The procedure is essentially the same. A number of "good" assemblies are tested in each band and averaged. A number of examples of each of several types of defects are similarly tested and the averages compared with those of the good units. For this purpose extreme cases should be used in order to show prominently which bands will bring out the greatest differences. Then after the bands have been selected, a number of barely acceptable and barely unacceptable units should be tested in these same bands to verify the selection and to establish standards.

All the above has been devoted to products in which the intensity of sound is used as the measure. As an example of one in which another quality than intensity of sound is used as a measure of precision, a study of grinding wheels might be mentioned. It has long been the custom of inspectors to judge hardness of the wheels by listening to their "ring" when tapped. None of the inspectors could tell exactly to what he

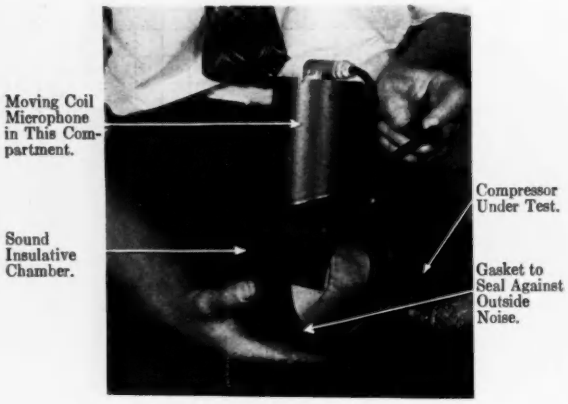

Moving Coil Microphone in This Compartment.
Sound Insulative Chamber.
Compressor Under Test.
Gasket to Seal Against Outside Noise.

FIG. 7. Special acoustic pick-up for routine test of small compressor units.

listened. Various acoustic properties of wheels previously classified as to hardness by laborious laboratory methods were measured. It soon became evident that over-all loudness was not the controlling factor. Oscillograms of the decay were made but if the decay time did correlate with hardness the difference in the oscillograms was so small as to not give much promise of a practical inspection method. On first attempt fair correlation was obtained on the basis of frequency measurements but there were some unexplained reversals in the data. The manufacturer with whom the study was being made improved his laboratory facilities for making hardness tests by the penetrometer method, and perfect correlation between frequency data and the corrected hardness readings was found.

This is just one example of the cases in which the use of acoustic tests has brought out incidental data which has resulted in improved design or manufacturing.

As more and more of these cases were encountered it became increasingly evident that a major part of the problem was in the acoustic pick-up system to fit each type of product. It was first thought that a vibration pick-up of some kind might be used in order to get around the problem of interfering plant noise, but results to date have been discouraging. It has not generally been possible to get consistent readings. This is probably caused partially by the influence of the human element in applying the pick-up to the product, and partially by small differences in dimensions and stresses between individual assemblies which cause differences in the vibrational pattern. These experiments are being continued, however, and we have some hope for a radically different type of pick-up applicable to certain types of assemblies in which we will measure the vibration as a whole unaffected, we believe, by variations in the components at specific points.

In discussing pick-up devices for this purpose it should be emphasized that we are comparing products which are identical physically. All we are trying to do is detect differences in all or part of two frequency response curves, and changes in the frequency response of the system introduced by the pick-up will not change the result since they will affect both curves equally. Some of the data shown are taken with pick-ups which affect the shape of the curve and are not the kind of data we would use in the development laboratory.

Results using acoustic pick-ups or microphones of various types have been encouraging. In some cases the product to be tested produced a high enough noise level or the manufacturer had a sufficiently good sound-proof room that it was possible to test with exposed microphones. In other cases it is sometimes possible to design an acoustic pick-up system which is not affected by plant noise. Some of the general types of pick-ups applicable under different circumstances may be of interest.

(1) *Exposed microphone.*—As described above it can only be used in the case of products having a high noise level or where a sound-proof room is available. In all cases of acoustic pick-ups the moving coil microphone is used to insure stability of response in that part of the system.

(2) *Shielded microphone.*—It is sometimes possible to build the microphone into a hood or acoustic coupling unit of some type so as to exclude the plant noise and pick up the noise from one or more surfaces of the product under test. Fig. 7 shows one such pick-up used in test of a small compressor. The compressor, of rotary type, is housed in the cylindrical casting at the center of the picture. The pick-up in the operator's hands in the picture is slipped over the compressor and clamped into place. A gasket

inside the microphone housing excludes the plant noise and permits measurement of the acoustic energy radiated by the end of the compressor shell. Sound isolation is sufficiently good that no error is caused by the operation of adjacent units on the test bench although spaced only a few inches away. Fig. 8 shows the average frequency response of a group of unacceptable units as compared to a satisfactory one. In this case, since there was only one type of compressor to be tested, it was preferable to design a special filter instead of using a flexible set and, on the basis of the above curves, a 2500 cycle high-pass was selected.

(3) *Muffler type pick-up.*—In some cases when not a very high suppression of the frequencies of plant noise to which an instrument is sensitive is required, the product can be passed through an insulated tunnel or muffler with the microphone located near its center. This constitutes an exacting mechanical and acoustical design problem as the means of moving the product must not itself produce interfering noise. A case of this type is illustrated in Fig. 9, which shows a fully automatic test equipment for detecting solder pellets in cans of liquids. The cans are rolled by gravity through the tunnel, passing the microphone near the center.

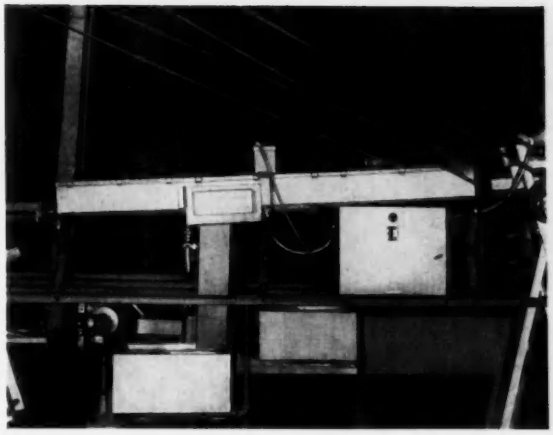


FIG. 9.

(4) *Sound-proof box.*—In a somewhat limited class of products routine tests can be made in a sound-proof box containing the product and the microphone only. This method does not lend itself so well to products having prominent low frequency components because of the difficulty

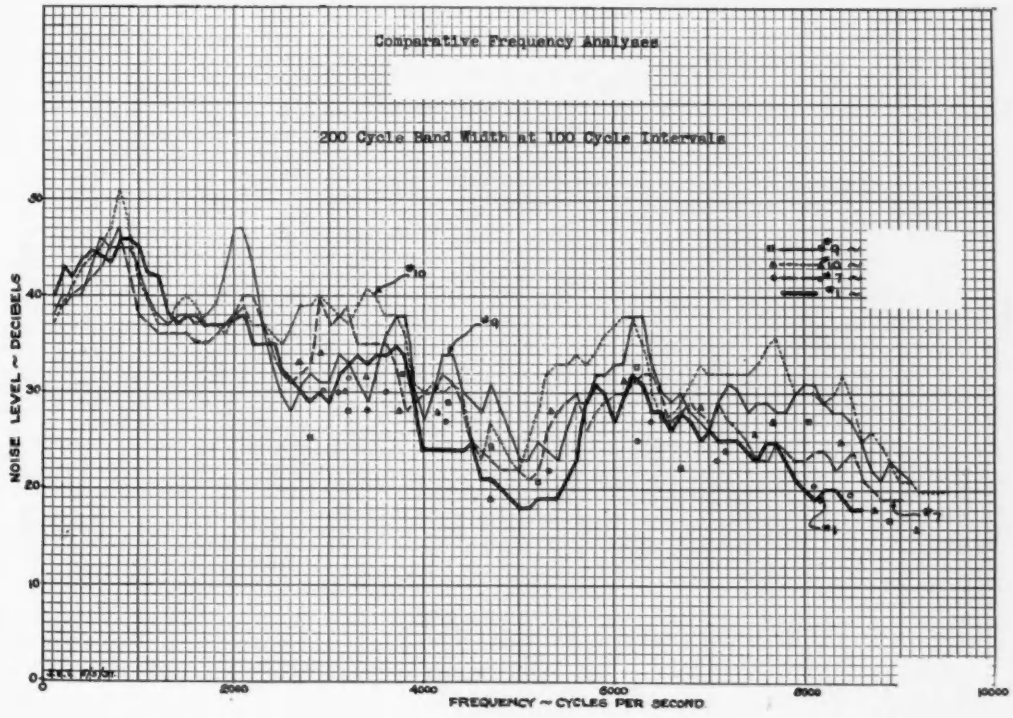


FIG. 8.

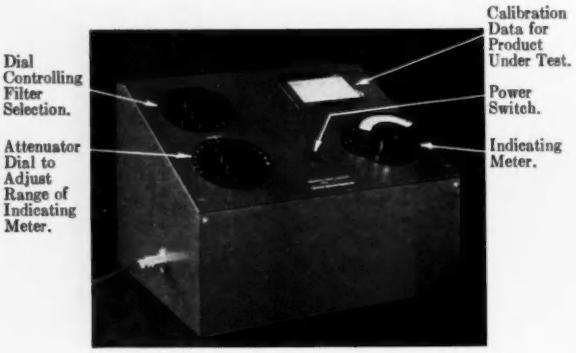

Dial Controlling Filter Selection.
Attenuator Dial to Adjust Range of Indicating Meter.
Calibration Data for Product Under Test.
Power Switch.
Indicating Meter.

FIG. 10. Industrial noise analyzer.

of obtaining high reduction of low frequencies in a small enclosure, also the effect of the enclosure on the acoustic performance of the product.

Figure 10 shows one version of a test instrument having the features discussed. The dial at the top controls the filter selection and the bottom dial the attenuator selection.

Success to date with applications of the above principles to various types of products indicates that the problem is now essentially an economic one rather than an engineering one. When the cost of special design can be justified it appears possible and practicable to design a pick-up and testing system for any product in which the acoustic output is itself of significance or can be used as a measure of mechanical precision.

The Frequency Ratios of the Tempered Scale

CHARLES WILLIAMSON
Department of Physics, Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received June 16, 1938)

A COURSE in acoustics for music students usually includes a consideration of the equally tempered scale. An experimental demonstration of the differences between just intonation and equal temperament is conveniently made with suitable tuning forks. The underlying theory, on the other hand, is likely to present mathematical difficulties. It is not here implied that typical music students lack the ability to understand twelfth roots and decimal fractions; on the contrary, many of them have learned what is needful about these things and could revive the information if they were asked to do so. However, it is questionable whether the requisite time and effort would be given ungrudgingly. Common fractions are more likely to be understood by the layman than are decimal fractions, because he has ordinarily had more occasion to use the former than the latter. Moreover, the student will have learned to represent the just scale with the aid of common fractions immediately before considering the tempered scale, and will thus be prepared for a similar representation of the latter. If, then, it were possible to explain the difference between tempered and just scales with the aid of common fractions, there ought to be some advantage in doing so.

Helmholtz[1] gives the following approximations to the incommensurable ratios of the tempered scale:

89/84	449/400	44/37	63/50	303/227	140/99
433/289	100/63	37/22	98/55	168/89	

While these are not exact, they are far more accurate than are the ratios actually found in a well-tuned piano or other keyboard instrument. They are also more precise than the four-figure decimal equivalents often cited. However, there is an objection to their use without modification.

In a recent article Harker[2] has furnished the decimal equivalents to eight significant figures, thus making it possible to calculate by the method of continued fractions[3] more expeditiously than would otherwise be possible. By this means it has been discovered that Helmholtz has given one of the convergents in only seven of the eleven cases. In three of the four cases where he has not done so, the nearest convergent gives a better approximation than the fraction appearing in his list. It is therefore suggested that the convergents be used in all cases.[4] The table would then read:

89/84	55/49	44/37	63/50	295/221	99/70
442/295	100/63	37/22	98/55	168/89	

It will be noted that the product of any pair of these fractions chosen symmetrically with respect to the middle one—99/70—gives exactly 2. This advantage is not possessed by some of those in Helmholtz' list.

TABLE I.

EXPONENT OF 2	CONTINUED FRACTION
1/12	(1, 16, 1, 4, 2, 7, ···)
2/12	(1, 8, 6, 31, 1, ···)
3/12	(1, 5, 3, 1, 1, 40, ···)
4/12	(1, 3, 1, 5, 1, 1, 4, 1, 1, 8, ···)
5/12	(1, 2, 1, 73, 11, ···)
6/12	(1, 2)
7/12	(1, 2, 147, 5, ···)
8/12	(1, 1, 1, 2, 2, 1, 3, 2, 3, 1, 3, ···)
9/12	(1, 1, 2, 7, 81, 3, ···)
10/12	(1, 1, 3, 1, 1, 2, 1, 1, 15, ···)
11/12	(1, 1, 7, 1, 9, 1, 15, 9, ···)

[1] *Sensations of Tone*, translated by Ellis (Longmans, Green and Co., 1912), p. 454.

[2] G. F. Herrenden Harker, J. Acous. Soc. Am. **8**, 243 (1937).

[3] *College Algebra*, Metzler, Roe and Bullard (Longmans, Green and Co., 1908), Chap. XIII. *Textbook of Algebra*, Chrystal (A. and C. Black, 1906), Part II, Chap. XXXII.

[4] The author does not claim originality for the use of these convergents. 55/49 appears explicitly in U. S. Patent 1,259,691 (1919) by T. Cahill; 99/70 implicitly in U. S. Patent 1,956,350 (1934) by L. Hammond; 295/221 and 442/295 may also be given in the literature, though the author has not seen them.

TABLE II.*

EXPONENT OF 2	CONVERGENTS
1/12	17/16, 18/17, **89/84**, 196/185, ···
2/12	*9/8*, **55/49**, 1714/1527, ···
3/12	6/5, 19/16, 25/21, **44/37**, 1785/1501, ···
4/12	4/3, *5/4*, 29/23, 34/27, **63/50**, 286/227, ···
5/12	3/2, *4/3*, **295/221**, 3249/2434, ···
6/12	3/2, 7/5, 17/12, 41/29, **99/70**, 239/169, ···
7/12	*3/2*, **442/295**, 2213/1477, ···
8/12	2, 3/2, 8/5, 19/12, 27/17, **100/63**, 227/143, ···
9/12	2, *5/3*, **37/22**, 3002/1785, ···
10/12	2, 7/4, 9/5, 16/9, 41/23, 57/32, **98/55**, 1527/857, ···
11/12	2, *15/8*, 17/9, **168/89**, 185/98, ···

* *Italics*, justly intoned intervals; **bold-face**, recommended equivalents for equally tempered intervals.

TABLE III.*

EXPONENT OF 2	DECIMAL EQUIVALENT (REFERENCE 2)	FROM HELMHOLTZ (REFERENCE 1)	ERROR (PARTS PER MILLION)	NEAREST CONVERGENT	ERROR (PARTS PER MILLION)	NEXT CONVERGENT	ERROR (PARTS PER MILLION)
1/12	1.059 4631	89/84	57.2	89/84	57.2	196/185	3.4
2/12	1.122 4621	449/400	33.8	*55/49*	*11.7*	1714/1527	0.3
3/12	1.189 2071	44/37	15.1	44/37	15.1	1785/1501	0.1
4/12	1.259 9211	63/50	62.5	63/50	62.5	286/227	7.3
5/12	1.334 8399	303/227	28.5	*295/221*	*13.0*	3249/2434	0.1
6/12	1.414 2136	140/99	51.0	*99/70*	*51.0*	239/169	8.8
7/12	1.498 3071	433/289	24.8	*442/295*	*13.0*	2213/1477	0.2
8/12	1.587 4011	100/63	62.5	100/63	62.5	227/143	7.3
9/12	1.681 7928	37/22	15.0	37/22	15.0	3002/1785	0.1
10/12	1.781 7974	98/55	11.6	98/55	11.6	1527/857	0.2
11/12	1.887 7486	168/89	57.3	168/89	57.3	185/98	3.3

* *Italics* indicate proposed changes from Helmholtz' list.

TABLE IV. *Selected convergents having errors of similar magnitudes.*

CONVERGENT	ERROR (PARTS PER MILLION)
196/185	3.4
55/49	11.7
44/37	15.1
286/227	7.3
295/221	13.0
239/169	8.8
442/295	13.0
227/143	7.3
37/22	15.0
98/55	11.6
185/98	3.3

SAMPLE COMPUTATION

Convergents to $(2)^{2/12}$

Harker's approximation: 1.12246205

Successive divisions:

0.12246205)1.00000000(8
0.97969640

0.02030360)12246205(6
12182160

64045)2030360(31
192135

109010
64045

44965)64045(1

Continued Fraction:

$$1+\cfrac{1}{8+\cfrac{1}{6+\cfrac{1}{31+\cfrac{1}{1+\cdots}}}}$$

In simplified notation:

$$(1, 8, 6, 31, 1, \cdots)$$

First convergent: 1

Second convergent: $\frac{8+1}{8}=9/8$

Third convergent: $\frac{6(9)+1}{6(8)+1}=55/49$

Fourth convergent: $\frac{31(55)+9}{31(49)+8}=1714/1527$

Similar computations are summarized in the Tables I–IV.

From the student's point of view the transition from the exact relations of the just scale to the approximations of the equally tempered scale is easier to understand if two cases, say 2/12 and 5/12, are quoted in full from Table II.

The Harmonic Structure of Vowels in Singing in Relation to Pitch and Intensity

BARRETT STOUT
Northeast Missouri State Teachers College, Kirksville, Missouri
(Received June 19, 1938)

THE purpose of this investigation was to study the changes that take place in the harmonic structure, as revealed by the Henrici analyzer, of sustained vowels in singing, under two conditions, first, with changes in intensity when pitch is held constant and, second, with changes in pitch when intensity is held constant. Lewis[1] and Tuthill[2] have shown that quality changes occur with changes in intensity and with changes in pitch. In those studies no attempt was made to isolate the effect of the two factors, pitch and intensity. The results, therefore, did not permit the determination of the effect of each of these two factors, independent of the other. It was hoped that the present study would throw some light upon the independent effect of these two factors on the harmonic structure of the sustained vowels "Ah," "OO" and "EE" in singing.

PROCEDURE

(1) Selecting the vowels, pitches and intensities

The vowels "Ah," "OO" and "EE" were chosen as being most representative of all the vowel sounds, since they lie at the three corners of the vowel triangle.

In most previous studies of sustained vowels, low or fairly low pitches have been employed. If it is desired to study the singing voice, medium and high pitches as well as low should be used. The octave $D_2=146.8$ cycles per second to $D_3=293.6$ cycles per second probably includes the pitches most used by the male singing voice. For this reason D_2, A_2, and D_3 were chosen for this study. There was a second consideration in determining the pitches to be used. It was hoped by choosing pitches that are in the natural speaking range of the average male voice, that this study might be made comparable in some respects to a parallel study by Laase[3] in the quality of vowels in speech. It was found by preliminary tests that two of the pitches chosen for the present study, D_2 and A_2, were well adapted to the requirements of the other study.

An attempt was made to have the subjects sing each of the three pitches at two intensity levels approximately 10 db apart. It was soon discovered that this could not be done. A tone sung on D_2 at an intensity level that would be described by the musician as piano, could not be produced on pitch D_3 at so low an intensity. In other words, the lowest intensity possible on D_3 was stronger than the intensity on D_2 that would be described as piano. This difficulty suggested the desirability of determining the functional singing area of each of the three subjects used in this study. This area may be described by a graph of intensity against frequency on which the tones that a singer can produce fall in an enclosed part of the total area. Such graphs were obtained in an acoustically treated room and with the aid of a microphone and its amplifying system terminated by a thermal voltmeter.[4] The subject stood with his lips approximately 12 inches from the microphone. He sang the loudest musically acceptable tones on the vowel "Ah" that he could produce, on all the pitches of the diatonic scale of F, from $F_1=87.3$ cycles per second, to $A_3=440$ cycles per second. This included his entire pitch range. For each tone so produced, readings were taken from the thermal voltmeter. The subject then sang the softest musically acceptable tones on the same pitches that had been used for the loud tones. Readings were taken from the volt-

[1] Don Lewis, "Vocal Resonance," J. Acous. Soc. Am. 8, 91 (1936).
[2] C. E. Tuthill, "Timbre and Sonance Aspects of the Sustained Vowel," "O," (Thesis), State University of Iowa, 1936.
[3] Leroy Laase, "The Effect of Pitch and Intensity on the Quality of Vowels in Speech," Ph.D. Thesis, 1937, State University of Iowa.
[4] This system consists of a Brush cell type crystal microphone with its associated amplifying system. The output of the microphone is amplified by a four-stage amplifier. The output of this amplifier is distributed to the recording amplifier and the amplifier for the oscillograph. A calibrated voltmeter using a thermocouple galvanometer in its output circuit was bridged across the output of the four-stage amplifier.

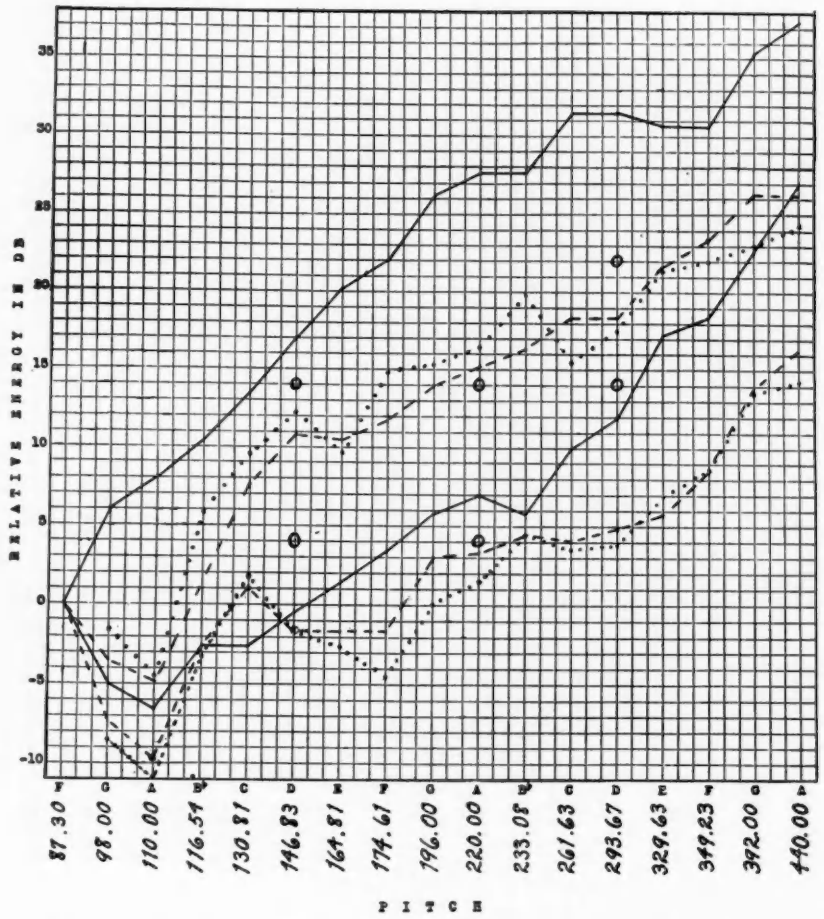


FIG. 1. The functional singing area for subject CO throughout his entire pitch range on the vowel "Ah." The solid lines enclose the area for the vowel "Ah," the dotted lines the vowel "OO" and the dot-dash lines for the vowel "EE." The circles indicate the intensities chosen for study.

meter for these tones. From these two sets of readings, the difference in energy in db between the loudest and softest tone possible on each pitch was determined. This same procedure was followed for the vowels "EE" and "OO." The results of these measurements for the subject CO are given in Fig. 1. The difference between the softest and loudest tones are plotted here from an arbitrary zero reference level.

It will be noticed that the curves for the loudest and softest tones meet at F_1. The subject could sing this pitch with just one intensity, and accordingly there was no difference in intensity between the softest and loudest tones possible at this pitch level. At pitch G_1, only one step higher, however, he was able to make a difference of 11 db between his softest and loudest tones on the vowel "Ah," a difference of 3.5 db on the vowel "EE" and a difference of 5.4 db on the vowel "OO." It will be observed that the curves for the vowel "OO" do not extend to a lower pitch than G_1. This subject could not sound the pitch F_1 on the vowel "OO." The curves for this vowel would meet somewhere between G_1 and F_1. It may be said, then, that the three pairs of curves for the three vowels "Ah," "EE," and "OO" outline the functional singing area for this subject on these vowels. It should be noted in passing that areas such as this vary considerably from time to time.

On the basis of this information the intensities to be used in this study were selected. The circles in Fig. 1 indicate the intensities chosen for the subject CO in relation to his intensity limits on

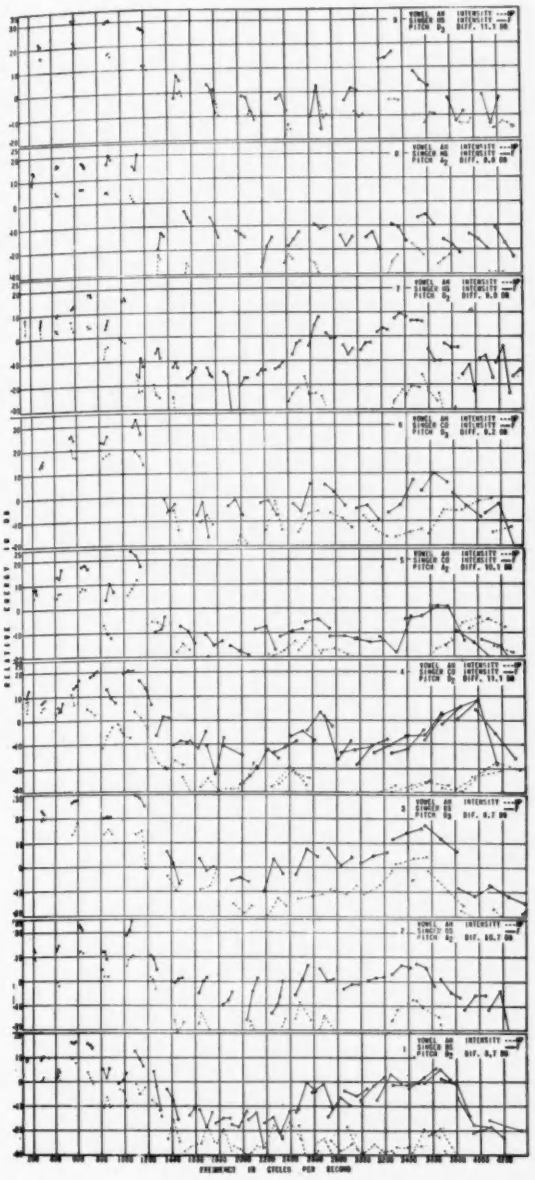


FIG. 2. Graphs showing changes in harmonic structure of the vowel "Ah" with changes in intensity, pitch constant. The large dots represent values for the forte tone and the small dots the mezzo piano tone. Values for the same partial in the three waves analyzed from one tone are connected by straight lines to emphasize intensity changes and outline contours.

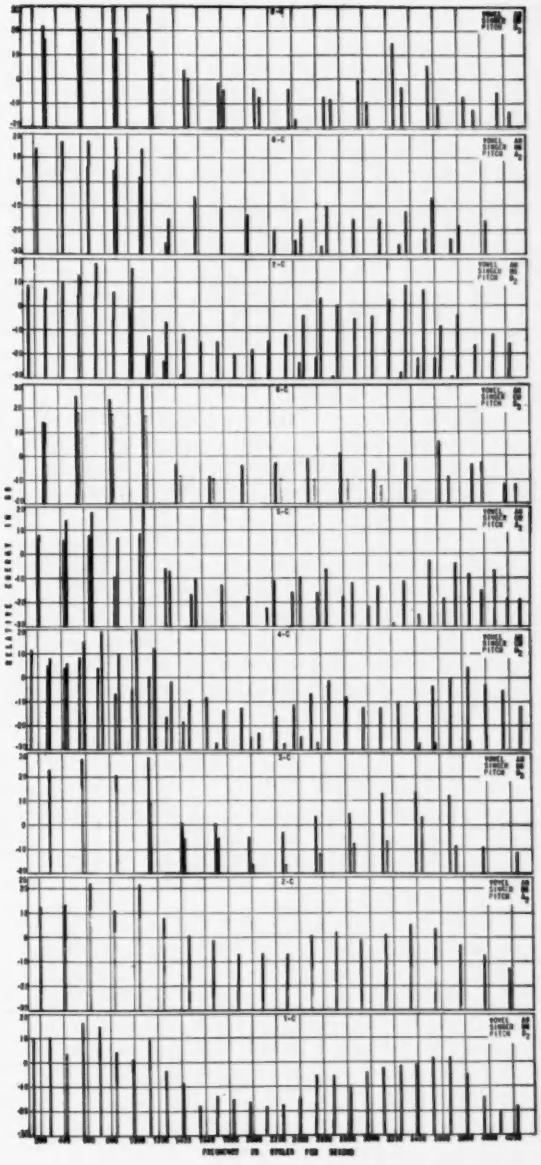


FIG. 2C. Graphs showing the same effect as Fig. 2, but with a single composite value for the three values for any partial. The pitch value is represented by the geometric mean of the three pitch values for any partial in Fig. 2. The intensity value is represented by the geometric mean of the three intensity values for any partial in Fig. 2.

the three pitches used. The information concerning the limits of the voices of the three subjects, with respect to the intensity range on the pitches selected, indicated that it would be well to use only two intensities on each pitch since it would be impossible to select a greater number of intensities that would come within the functional singing range of the subjects for all three vowels and still have as large an intensity difference between them as was desired.

Since it was the purpose of this investigation to study the effect of both pitch and intensity on vowel structure, it was necessary to choose intensities so that in some cases intensity might

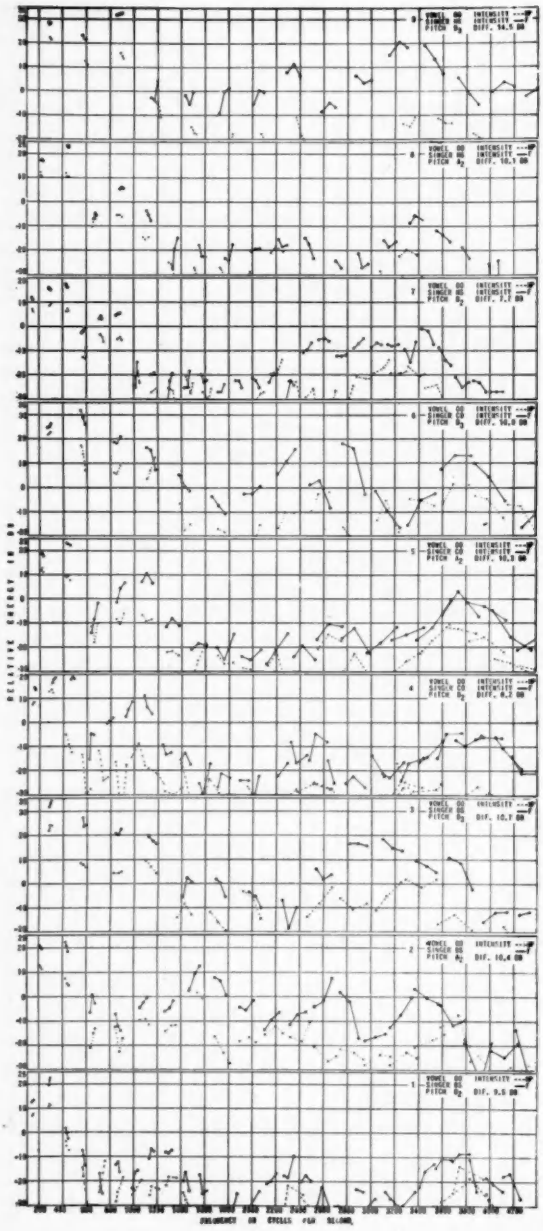


FIG. 3. Graphs showing changes in harmonic structure of the vowel "OO" with changes in intensity, pitch constant. The large dots represent values for the forte tone and the small dots the mezzo piano tone. Values for the same partial in the three waves analyzed from one tone are connected by straight lines to emphasize intensity changes and outline contours.

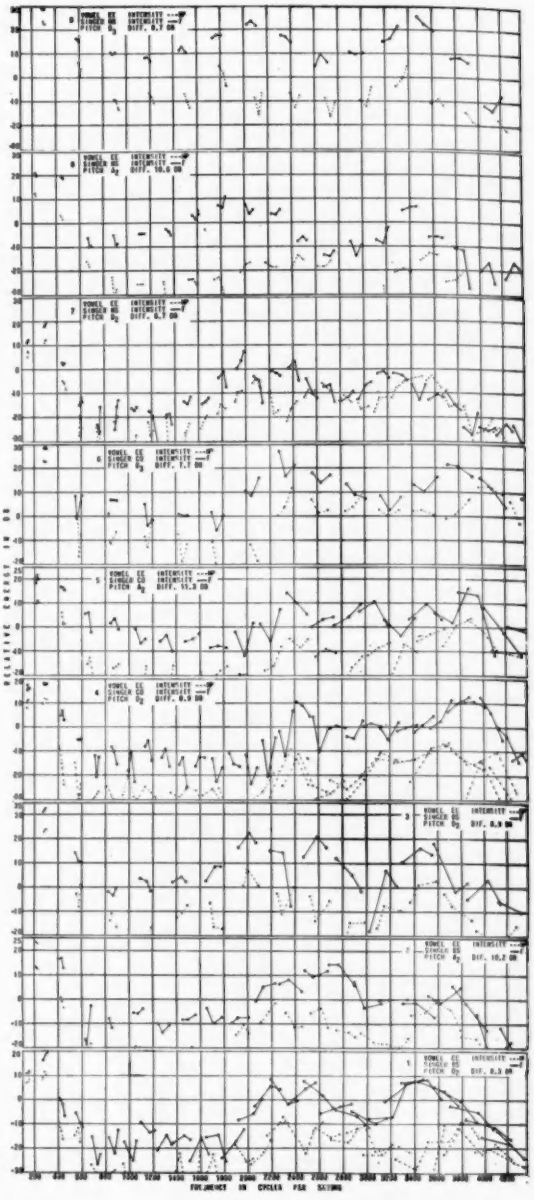


FIG. 4. Graphs showing changes in harmonic structure of the vowel "EE" with changes in intensity, pitch constant. The large dots represent values for the forte tone and the small dots the mezzo piano tone. Values for the same partial in the three waves analyzed from one tone are connected by straight lines to emphasize intensity changes and outline contours.

remain constant at different pitches. It will be observed from Fig. 1 that the intensities selected for the louder tone on D_2, the louder tone on A_2, and the softer tone on D_3 are all equal. It may be observed that some of the intensities chosen lie outside the intensity limits for some of the vowels. For example, the intensity for the louder tone on D_2 lies slightly above the upper limit for the vowels "EE" and "OO." This is not as contradictory as it appears from the graph, because these limits are somewhat flexible and

it was found in practice that the subject could reach the intensity chosen.

(2) Selecting and training the subjects

Three trained male singers were selected as subjects. One of these singers was head of the voice department in the State University of Iowa; another was teacher of voice in the Northeast Missouri State Teachers College; the third was a graduate student of considerable training and singing experience.

(3) Obtaining oscillograms and phonograph recordings

The apparatus was set up just as it was for graphing the functional singing area. The subject stood with his lips about 12 inches from the microphone. He was instructed to sing a specified vowel at a specified pitch with his best musical quality. As soon as the tone had gained the required intensity as indicated by the thermal voltmeter, a signal was given to the photographer and an oscillogram of sufficient length to include at least one complete vibrato cycle, was taken. A phonograph recording was made simultaneously with the oscillogram for purposes of checking.

(4) Selecting, measuring and analyzing the waves

Three waves were selected from each oscillogram, one from the trough, one from the slope and one from the crest of a pitch vibrato cycle, and analyzed by means of an Henrici harmonic analyzer. Corrections were made for differences in enlargement of the waves due to differences in fundamental frequencies of the three waves. Corrections were also made for oscillograph loss, calibration of the system having been made at the time the oscillograms were made. The waves from tones at pitch D_2 were analyzed to 30 components, those from pitch A_2 to 20 components and those from D_3 to 15 components. In all cases this included all components up to 4200 cycles. A few tones were analyzed to higher frequencies, but very little energy was found above 4200 cycles. This fact, together with the fact that previous studies had shown that there was very little energy in vocal tones above 4000 cycles, indicated that it was unnecessary to carry the analysis to higher components. The results of the analysis are shown in Figs. 2 to 4.

(5) Computing, tabulating and graphing the results

The method of graphing the results of analysis used by Lewis was adopted for this study. Frequency is plotted on a linear scale on the abscissae and relative energy in db on the ordinates. The zero reference level is comparable throughout this study from graph to graph. A range of 40 db for any one tone is plotted, but because of the fact that two tones with a difference of approximately 10 db in intensity are plotted on the same graph, it was necessary to let each graph cover a range of about 50 db.

Referring to graph No. 1 at the bottom of Fig. 2, it will be observed that both the mezzo piano tone and the forte tone[5] for pitch D_2, vowel "Ah," subject BS, are represented here. The forte tone is represented by the larger dots connected by solid lines. The mezzo piano tone is represented by the smaller dots connected by broken lines. Each group of three connected dots represents either the fundamental or one of its harmonics, the first group at the left representing the fundamental. The three dots of any group correspond to the intensity level values for the same partial of the three waves from one tone. These three dots are connected by straight lines to emphasize intensity changes and show general contours of the major intensity areas. The first dot at the left in each group of three dots repre-

TABLE I–A. *Increase in intensity (in db) in the frequency regions below and above 1800 cycles, with an increase of approximately 10 db in the total intensity of the tone.*

		"Ah"		"OO"		"EE"	
SUBJECT	PITCH	BELOW	ABOVE	BELOW	ABOVE	BELOW	ABOVE
HS	D_3	11.0	14.4	11.2	27.0	6.8	23.3
CO	D_3	9.2	9.1*	7.9	14.5	5.9	12.3
BS	D_3	9.3	12.9	11.1	15.3	8.4	16.5
HS	A_2	9.7	13.3	10.2	11.0	10.4	20.9
CO	A_2	10.1	6.0*	10.3	11.4	10.7	13.3
BS	A_2	10.5	16.8	10.3	15.0	10.7	13.9
HS	D_2	9.4	26.3	7.6	12.8	6.7	5.8*
CO	D_2	10.9	23.0	8.1	17.4	6.5	19.1
BS	D_2	7.0	21.5	9.6	15.3	7.2	15.2

[5] The designations "mezzo piano" tone for the tone of lesser intensity and "forte" tone for the tone of greater intensity are used throughout this paper for convenience. Perceptually these terms do not always describe the tones adequately. The reason for this can readily be observed from Fig. 1.

sents the energy value of the wave selected from the trough of the vibrato cycle; the middle dot represents the wave from the slope and the third dot the wave from the crest of the vibrato cycle. The three values for the fundamental component are clustered at the extreme left between 137 and 146 cycles. They show a variation in relative intensity of less than one decibel. It appears from this graph that the fundamental component of the mezzo piano tone is not represented. This is because the values are so nearly the same as for the forte tone that the difference cannot be represented on so small a scale. In the second component, however, there is a difference of approximately 9 db in the relative intensity level for the two tones. In the third component the values are again close together. By representing on one graph the two tones of different intensities at the same pitch, one is able more readily to observe structural changes with increase in intensity.

Results

(1) The effect of intensity on harmonic structure

For convenience in studying the changes in the relative intensity of the high and low partials of the tone we may divide the spectrum at the frequency 1800 cycles. This frequency was chosen because in all the graphs except one the region around 1800 cycles appears as a region of relatively low intensity. The changes in relative intensity of the high and low partials in the vowel "Ah" may be observed from the nine graphs of Fig. 2. The most prominent change is a greater increase in the intensity of partials in the frequency regions above 1800 cycles than in regions below this point. The same change in the relative importance of the two regions may be observed in the graphs for "OO" and "EE," Figs. 3 and 4. A quantitative statement of the increase in intensity (in db) in the frequency regions below and above 1800 cycles with an increase of approximately 10 db in the total intensity of the tone, is given in Table I-A. The enhancement of the relative importance of the high frequencies with an increase in intensity is greatest at the low pitch for "Ah," greatest at the high pitch for "EE" and about the same for all pitches for the vowel "OO." This trend toward an increase in the relative importance of the high frequency regions, though quite marked, is not entirely consistent. The 27 graphs show 3 exceptions. (These exceptions are indicated in Table I-A by an asterisk.) In the same number of cases, Laase[3] found no exceptions to this trend. These three exceptions are probably due to the inability to control all the factors involved in producing the tone. The existence of so few exceptions and the magnitude of this change in relative importance of the high and low frequencies with increase in intensity, in the other instances, are quite significant of the operation of a general law.

The fundamental in the case of the vowel "Ah" remains practically unchanged with an increase in total energy of the tone; it increases some in the case of vowel "OO" and still more in the case of vowel "EE."[6]

There appears to be no consistent change in the frequency location of the major intensity areas with an increase in intensity. For only one subject and one vowel is there a consistent change evident. In the case of subject CO in the vowel "Ah" the highest intensity area shifts to a lower frequency region with an increase in intensity.

Figure 2*C* is illustrative of another method of representing the same data. The graphs in this figure are composites of those in Fig. 2. The solid bars represent the forte tone and the open bars represent the mezzo piano tone. Frequency is represented by the geometric mean of the three frequencies in Fig. 2 for any component. Relative intensity is represented by the geometric mean of the three intensity values for any component. In cases where the frequency is the same for the two tones, the solid bar representing the forte tone is superimposed upon the open bar. The point where the open bar ends and

[6] This appears to be a contradiction of the findings of Tuthill (reference 2) for the vowel "O." But in reality it is in agreement. Tuthill states that with an increase in the intensity of the total tone, the fundamental component undergoes a decrease in relative energy. Another way of stating it would be that some other partials among the first six assume much greater importance with an increase in energy of the total tone, while the fundamental remains fairly constant. Tuthill's graphs, because the zero reference level shifts in absolute value from graph to graph, give the impression, upon superficial examination, that the fundamental component becomes less in absolute value with an increase in the energy of the total tone.

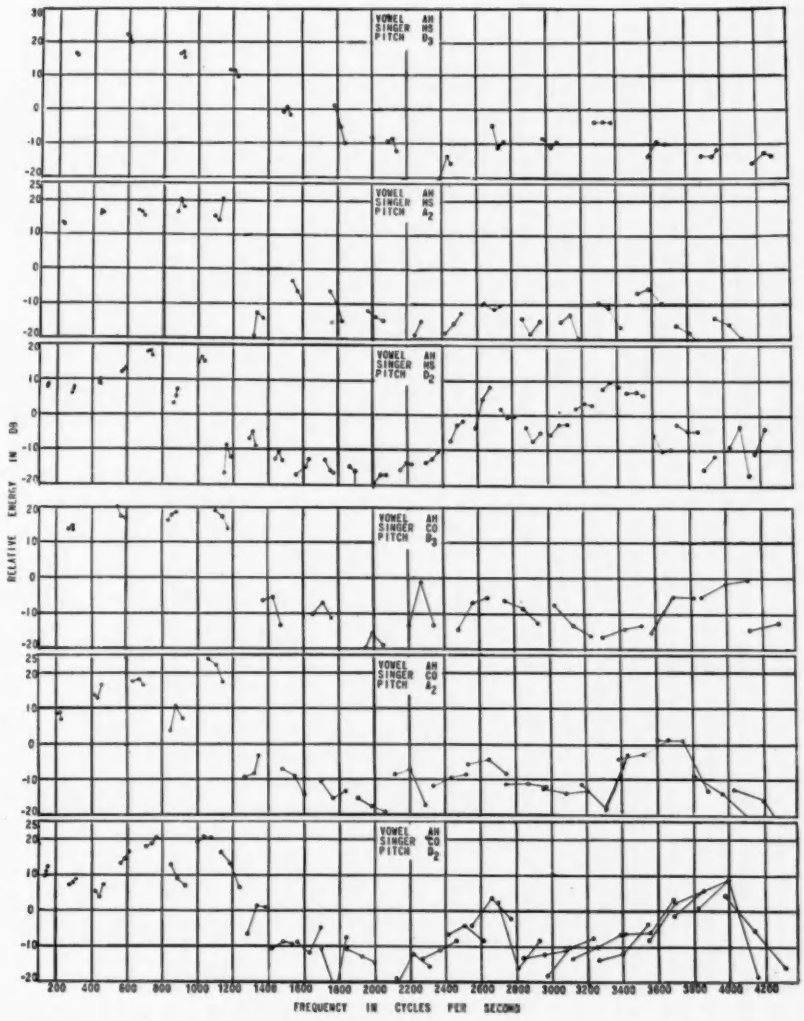


FIG. 5. Graphs showing the effect of changes in pitch on the harmonic structure of the vowel "Ah" for two subjects. The three values for the same partial in three waves selected from the same tone are connected by straight lines to emphasize intensity changes and outline intensity areas.

the solid bar begins is the value for the mezzo piano tone. The top of the solid bar represents the value for the forte tone. Usually there is a slight difference in the frequencies of the two tones represented on one graph. This distributes the bars so that in most cases they do not interfere with one another.

(2) The effect of pitch on harmonic structure

For the study of the effect of pitch upon harmonic structure, certain tones of different pitch were selected which were known, from the results of analysis, to be of practically the same intensity. It will be recalled that in selecting the intensities to be used in this study, the attempt was made to have the forte tone on D_2, the forte tone on A_2, and the mezzo piano tone on D_3 the same intensity. The analysis showed that this was not always accomplished. However, it was accomplished in enough instances to provide a satisfactory study. The greatest difference between the tone of least intensity and the tone of greatest intensity in any group being compared in this part of the study was 2.2 db. The smallest difference was 0. So we may say, for practical purposes, that the intensity of the tones in any group being compared for the study of the effect of pitch on harmonic structure was

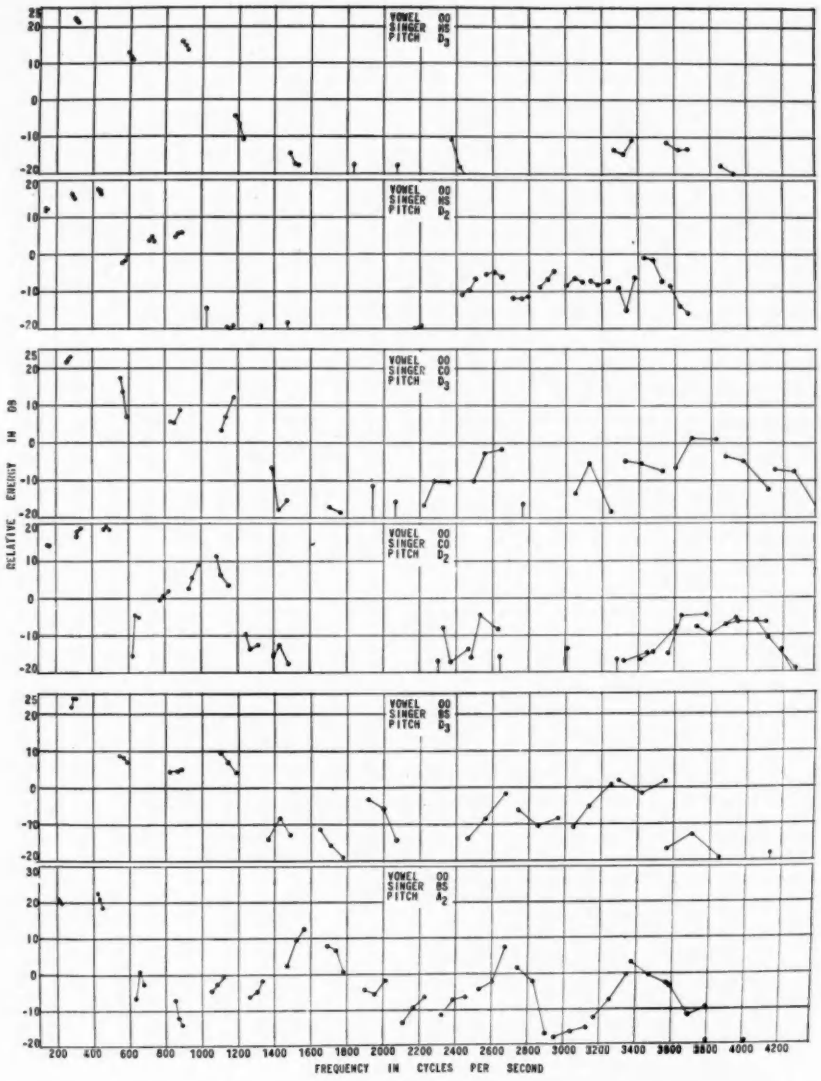


FIG. 6. Graphs showing the effect of a change in pitch on the harmonic structure of the vowel "OO" for three subjects. The three values for the same partial in three waves selected from the same tone are connected by straight lines to emphasize intensity changes and outline intensity areas.

held constant. Fig. 5 represents the results of analysis of three tones on the vowel "Ah," on the three selected pitches with intensity held constant,[7] for the subject CO in the lower half of the figure and for subject HS in the upper half.[8] One definite trend may be observed. As the pitch is raised, energy is shifted from the high frequency regions to the low.[9] The same trend exists for the other two vowels, Figs. 6 and 7. This trend is shown quantitatively in Table I-B.

This trend is not as strongly indicated as is the trend that was observed with a change in

[7] These tones would probably be described perceptually as forte for the lowest pitch, mezzo forte for the middle pitch, and piano for the highest pitch.

[8] Comparisons should not be made from subject to subject. Each subject must be studied separately on account of individual differences.

[9] This trend might be stated inversely: As the pitch is lowered, intensity constant, the intensity of the high frequencies increases. It is interesting to observe, then, that in all cases when the intensity in the high frequencies increased, regardless of whether the accompanying change was one of pitch or intensity, the singer reported greater effort in producing the tone, and a listener reported a louder tone.

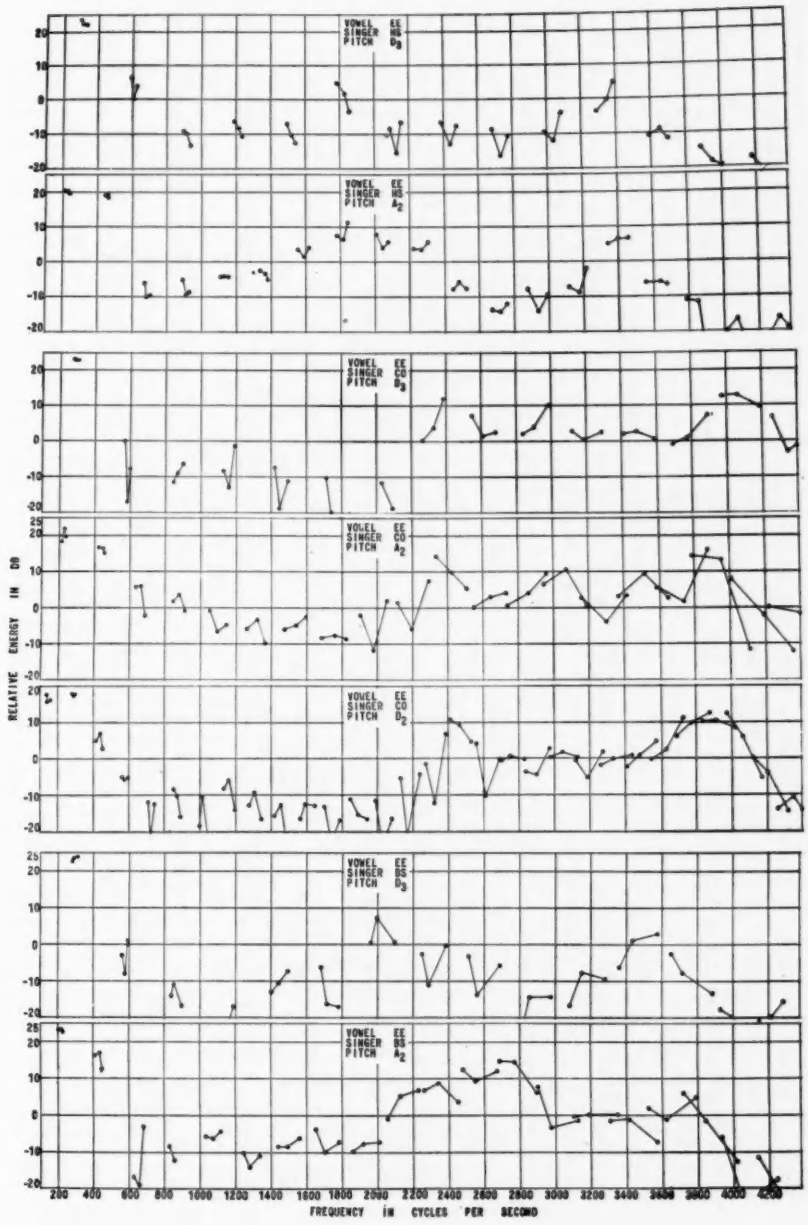


FIG. 7. Graphs showing the effect of a change in pitch on the harmonic structure of the vowel "EE" for three subjects. The three values for the same partial in three waves selected from the same tone are connected by straight lines to emphasize intensity changes and outline intensity areas.

intensity. There are three exceptions in the eleven cases studied. These exceptions are indicated in Table I-B with an asterisk. Laase[3] found this same trend and about the same proportion of exceptions.

The two intensity contours at frequencies 600–800 and 1000–1200 for the vowel "Ah" appear to become less definitely two contours, the higher the pitch, until on the highest pitch they appear to have merged into one large contour. This, of course, may be because of the fact that the fundamentals of the higher pitches do not happen to have partials at the proper frequencies to outline the two separate contours.

In the case of all three vowels, the fundamental appears to absorb a considerable part of this shifted energy with a rise in pitch.

There appear to be no consistent changes in the frequency location of the major intensity areas with a rise in pitch.

Conclusions

The results of the present study indicate (1) that the most important change in the harmonic structure of the vowels "Ah," "OO" and "EE" which accompanies an increase in intensity, pitch remaining constant, is an enhancement of the relative importance of the partials lying above the frequency 1800 cycles; (2) that this enhancement is greatest at the low pitch for the vowel "Ah," greatest at the high pitch for vowel "EE" and about the same at all three pitches studied for vowel "OO;" (3) that the increase in the fundamental which accompanies an increase in total intensity is very slight for the vowel "Ah," considerably more for the vowel "OO" and greatest for the vowel "EE." There were other changes which did not appear consistent enough to warrant any general statements.

The results of the study also indicate (1) that the most important change in the harmonic structure of the vowels "Ah," "OO," and "EE" that occurs with a rise in pitch, intensity remaining constant, is a decrease in the relative intensity of the partials lying above the frequency 1800 cycles; (2) that the contours outlining the intensity areas at frequencies 600–800 and 1000–1200 for the vowel "Ah" appear to become less definitely two contours, the higher the pitch, until at the highest pitch they appear to have merged into one large contour; (3) that in the case of all three vowels, the fundamental appears to absorb a considerable part of the energy which has shifted from the high frequency regions to the low.

That there is no consistent change in frequency location of the major intensity areas with either an increase in total intensity or a rise in pitch is quite definitely indicated by the results of this study.

Table I–B. *Relative intensity (in db) in the frequency regions below and above 1800 cycles at different pitches when the intensity is held approximately constant.*

		D_2		A_2		D_3	
Vowel	Subject	Below 1800	Above 1800	Below 1800	Above 1800	Below 1800	Above 1800
"EE"	HS			23.0	12.4	22.9	4.0
	CO	20.4	17.0	22.0	18.9*	23.0	15.6
	BS			23.8	16.8	23.6	7.2
"OO"	HS	20.3	2.8			23.0	−6.1
	CO	22.4	.9			23.2	4.4*
	BS			23.8	8.5	23.4	4.2
"Ah"	HS	21.6	13.1	23.2	−1.7	25.3	1.6*
	CO	24.6	11.2	24.0	3.8	24.7	2.5
	BS						

An Apparatus for Direct-Recording the Pitch and Intensity of Sound[1]

JÛICHI OBATA AND RYÛJI KOBAYASHI
Physics Department, Aeronautical Research Institute, Tokyo Imperial University, Tokyo, Japan

SINCE we described the essential part of this apparatus, the direct-reading pitch recorder,[2] about a year ago, we have improved various parts of the arrangement with the object of obtaining not only accurate results, but also of making it possible to follow much more rapid variations in pitch and intensity than has been possible heretofore.

In order to strengthen the fundamental tone, a valve with quadratic characteristic was used in the wave form adjuster, and in order that the input of the thyratron frequency meter shall be neither too large nor too small, a so-called symmetrical compressor was used instead of the simple volume controller of the former arrangement. In this connection, we should remark on the use of a compressor. In order to make the time-constant as small as possible, perfect symmetry should be maintained in the compressor, and on the other hand, since the use of a compressor inevitably introduces errors for the transient phenomena, better results are frequently obtained, in the case of a very large change in pitch, by omitting the compressor.

As to the direct-reading intensity recorder, which is often very necessary in studying music or speech, a number of devices[3] are already known as the level-recorder, all of which devices were carefully considered, resulting in our adoption of the one shown here as the most convenient for our purposes.

The electrical connections of the general arrangement are shown in Fig. 1, and a photograph of the complete assembly in Fig. 2.

As already stated in the previous paper, the pitch is indicated in logarithmic scale. In the present arrangement the intensity (or the effective value of the amplitude) is also indicated logarithmically by means of the combined effect of the symmetrical compressor and the characteristic of the valve in the final amplifier. The range of accurate indication of intensity or level extends about forty decibels.

A radio receiving set is also provided, by means of which graphs may be obtained for radio programs.

For marking the reference lines c, c^1, c^2, etc. on the graph, a separate "relaxation" circuit utilizing a tuning fork is provided (Fig. 3). Two c^2-forks, tuned to *international* pitch (517.3) and to *concert* pitch (523.3) are used, and with the aid of a single relaxation circuit and a change-

[1] The expenses incurred in the prosecution of the present study were defrayed from a subsidy granted the authors by Baron Kishichiro Okura, the complete apparatus being manufactured and marketed by the Japan Wireless Telegraph and Telephone Co., Ltd., Higashi-Osaki, Tokyo, Japan.

[2] J. Obata and R. Kobayashi, Proc. Imp. Acad. Tokyo **13**, 247 (1937); J. Acous. Soc. Am. **9**, 156 (1937).

[3] E. Meyer and L. Keidel, E. N. T. **12**, 213 (1935). E. C. Wente, E. H. Bedell and K. D. Swartzel, Jr., J. Acous. Soc. Am. **6**, 121 (1935). W. Holle and E. Lübcke, H. T. E. A. **48**, 41 (1936). H. G. Thilo and M. Bidlingmaier, Zeits. f. tech. Physik **17**, 558 (1936).

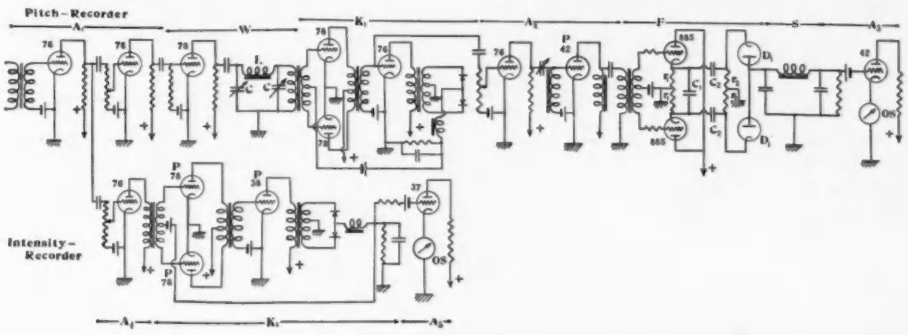


FIG. 1. Electrical connection.

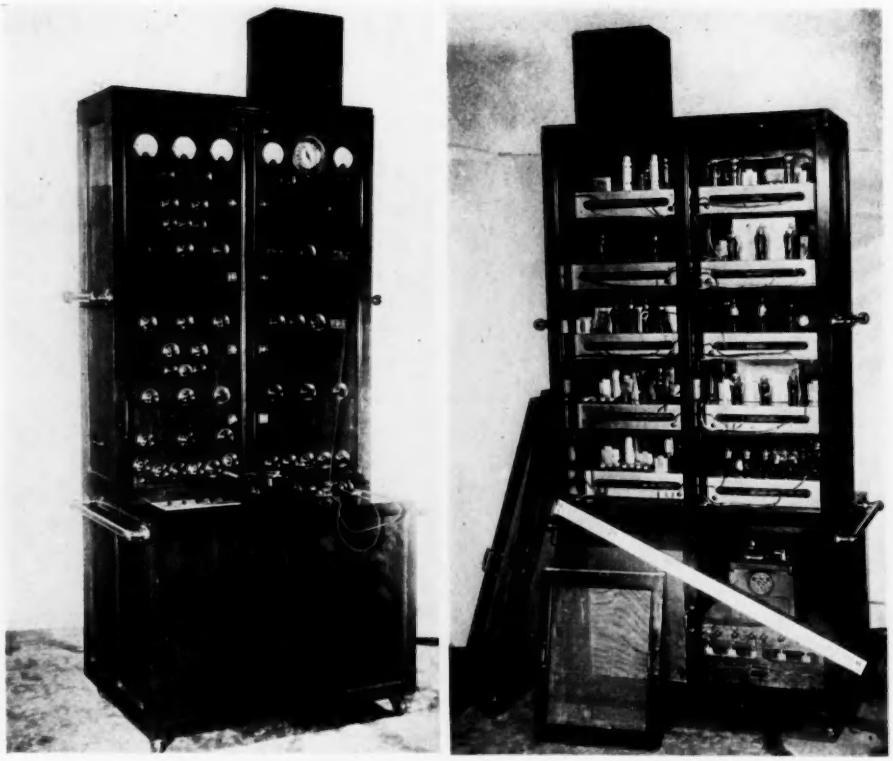


FIG. 2.

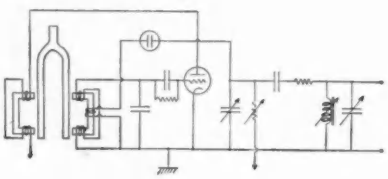
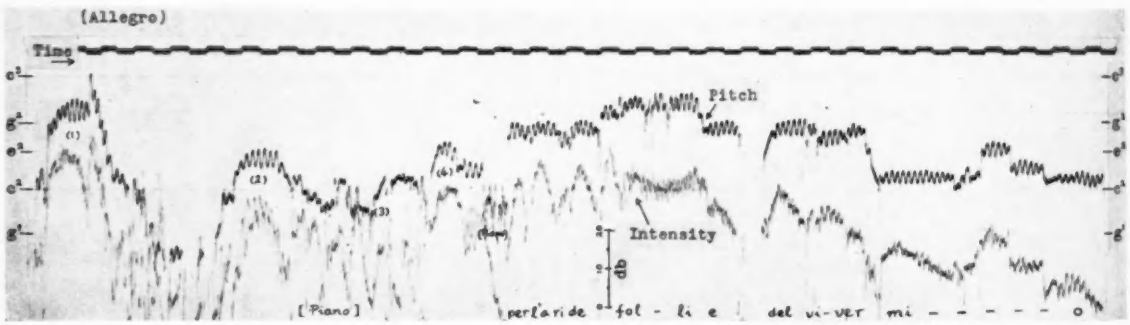


FIG. 3. Standard oscillator.

over switch, reference lines in either of the above pitches may be recorded.

As will be seen from the photograph, ample space is provided for all parts of the apparatus, which, however, has made the apparatus rather bulky, it being about 3 feet wide, 6 feet high, and 1 foot deep. Direct current sources are used for the most part of the circuits, the secondary



(1) (2) (3) (4)
Oh gioja ch'io non conobbi, esser amata a mando! E sdegnarla pos s'io

FIG. 4. Soprano solo by Eidé Norena, a part of the recitative preceding the aria "Ah, fors' è lui" in "La Traviata," Act I. Victor record RL–3–A.

batteries being kept in the case that forms the lower part of the apparatus. The use of eliminators may however be greatly increased, and if recording of pitch alone is required, the apparatus could be reduced to less than one-half the present size.

Figure 4 is an example of a pitch- and intensity-graph taken with the present arrangement, the thick line being the pitch-graph or the melody curve and the thin line the intensity-graph. The time scale was obtained by means of a simple electrical device, contained in a separate box (not shown in the photograph), utilizing a metronome; each dash in the graph representing one-half second.

Details will shortly be published in the *Proceedings of the Physico-Mathematical Society of Japan*.

Acoustical Society News

(The Editor will appreciate receiving items of news about scientific matters, honors or prizes given to members of the Society, etc. Address such notes to F. R. Watson, University of Illinois, Urbana, Illinois.)

THE next meeting of the Society will be held November 18–19 at Harvard University, Cambridge, Massachusetts. Professor F. A. Saunders is Chairman of the committee on program and arrangements. A second meeting will be held about May 1, 1939, in New York City, during which a celebration of the tenth anniversary of the Society is being arranged. A third meeting will be held in the Fall of 1939 at the State University of Iowa. Further announcements about the meetings will be sent to members by the Secretary.

A banquet was held on June 19 in honor of Professor G. W. Stewart, who is beginning his thirtieth year of service as head of the Department of Physics at the State University of Iowa. At that time, a portrait of Professor Stewart was presented to the University by colleagues and former students. Professor Stewart responded with an address on "The Human Values of Physics."

The U. S. Civil Service Commission announces examinations for various positions for physicists who will be employed in government service. The positions and salaries are as follows: junior physicist, $2000 a year; assistant physicist, $2600; associate physicist, $3200; physicist, $3800; senior physicist, $4600; principal physicist, $5600. Inquiries about details may be addressed to Mr. L. A. Moyer, Chief Examiner, U. S. Civil Service Commission, Washington, D. C.

Book Reviews

Sound Waves and Acoustics. M. Y. COLBY, Professor of Physics, The University of Texas. Pp. 350. Henry Holt and Co. Price $2.80.

This is an interesting textbook of intermediate difficulty, presenting the usual subjects of introductory acoustics with the aid of elementary mathematics, including a small amount of calculus. The treatment is up to date, especially in the sections on microphones, hearing, and supersonics. It is even mentioned that the inverse square law is of limited application in acoustics.

The style is such as will enlist the students' interest; for instance, he considers the possibility of warming his coffee by shouting into it, but after computing that the shout would have to be continued for 75 years in order that the absorbed acoustical energy should produce the desired effect, decides to drink the coffee cold. A generous number of problems and completions exercises is provided. A creditable book.

F. A. FIRESTONE
University of Michigan

Motion Picture Sound Engineering. Research Council of the Academy of Motion Picture Arts and Sciences. Pp. 534. D. Van Nostrand Co. Price $6.00.

This book is a sequel to *Recording Sound for Motion Pictures* which was published in 1931, and has been compiled since 1936 from stenographic notes of a series of lectures delivered to approximately 200 selected studio sound department employees by a number of lecturers.

Part I, consisting of the lectures constituting the Advanced Course in Sound Recording, was contributed principally by Fred Albin, L. E. Clark, John Hilliard, and Harry Kimball. Some of the chapter headings are as follows: Types of Film Recording, Noise Reduction, Microphones, Headphones and Loud Speakers, Phase Distortion, General Network Theory, Attenuation Equalizers, Wave Filter Theory, Dividing Networks for Loud Speaker Systems, Amplifier Circuits, and Rectifiers. While the theory underlying these subjects is presented at least in outline, special emphasis has been given to presenting the final results in such a form as to be most conveniently available for practical application. The sections on equalizer and filter design, particularly, contain a considerable number of curves presenting design data.

Part II, contributed by A. P. Hill, was the course in the Fundamentals of Sound Recording and is a comparatively elementary treatment of those branches of electrical engineering which are applicable in sound recording. It explains the decibel, the elementary theory of amplifiers, resonant circuits, vectors, resistance attenuation networks, etc.

The book is well done and will no doubt be the Bible for those sound technicians connected with the picture industry, howsoever, it contains much material which will be of interest to others working with sound recording or sound transmission. (Since this reviewer always has trouble with his spelling, he feels qualified to point out that the principle which he has always seen referred to as Thevenin's Theorem, is, throughout this book, called "Thevenim's Theorem.")

F. A. FIRESTONE
University of Michigan

Music and Sound. LL. S. LLOYD, with a foreword by Sir William Bragg. Pp. 181, Figs. 53. Oxford University Press, 1937. Price, $3.50.

This text has chapters on the pure scale, temperaments, chromatic notes, combination tones, consonance and dissonance, theories of harmony, the nature of sound, tone-quality, sensitivity of the ear, resonance, organ pipes, orchestral wind instruments, and string-vibration, together with 11 appendices giving tables and mathematical treatments. "The excuse for the existence of this book is that it approaches the subject from the point of view of the history of musical composition."

It is largely a brief review of the familiar ground covered by Helmholtz. The introduction states that the book is intended in the first instance for the use of students of music. In spite of this laudable aim, however, and in spite of its merit in being written from the musician's standpoint, it fails to be of great service to the average music student. Too difficult for understanding without more knowledge of mathematics than the music student possesses, while at the same time too brief for a text on the physics of sound, its principal worth seems to be as a quite readable introduction to Helmholtz and to the general field of acoustics, for the physics student with a musical bent or for the very exceptional musician with a physical background. Various topics, such, for example, as that of orchestral wind instruments, seem to be too summarily treated to be of great value to musician or scientist, particularly in view of the amount of space devoted to topics of less importance to practical music.

In addition to the chapters on scales and temperaments, perhaps the best sections are those treating of the quality of musical tones and of the vibration of strings, and analyzing, with helpful diagrams, the superposition of displacement curves and of traveling waves. In fact, the diagrams are uniformly excellent throughout the book.

The author's position with regard to just intonation seems not clear. He realizes the adaptability of the ear and the fact that the exact position of the scale degrees varies with their aesthetic significance. However, he takes for granted that "in all contrapuntal writing the intervals themselves, for harmonic reasons, would be the intervals of the pure scale," and that the pure scale "is still the natural scale for unaccompanied voices or strings." These are *a priori* statements which are open to question. He feels that even though orchestral music may be conceived as having a skeleton of equal temperament, it actually comes to life in performance with a more or less close approximation to just intonation. The recent study of violin performance by Greene has shown that the approximation is rather toward Pythagorean intonation than toward just.

The constant usage of "note" to mean a *tone*, instead of restricting it to its proper meaning of a printed symbol, is unfortunately not restricted to this author.

WILMER T. BARTHOLOMEW
Peabody Conservatory of Music

Hearing: Its Psychology and Physiology. STANLEY SMITH STEVENS AND HALLOWELL DAVIS. Pp. 489+xv, Figs. 166, 15×23 cm. John Wiley and Sons, Inc., New York, 1938. Price $4.50.

The purpose of the authors in writing this book was twofold: (1) "To provide the students of psychology, physiology, acoustics, and otology with an inventory of the recent discoveries in the psychophysiology of hearing," and (2) "To test the progress of the study of audition by casting up the balance in systematic form, taking stock of the gaps and deficiencies, and finding to what extent auditory research is able to yield a consistent point of view." In both phases of their purpose they have succeeded admirably. Their work will take its place beside Helmholtz' *Lehre von den Tonempfindungen* and Fletcher's *Speech and Hearing*.

As the reader first glances through the book he is struck by the large number of excellent graphs, drawings, photographs and oscillograms, and by the unity and continuity of the text, achieved by appropriate division of the chapters into sections and by omission of footnotes. All papers are referred to in the text by author's name; the full references (about 350 of them) are given in the alphabetized table at the end. Certain specialized subjects—particularly mathematical developments—are collected in the appendixes. The carefully compiled name and subject indexes make the material readily available for reference purposes. A more thorough reading reveals the value of the glossary of acoustical, physiological, and psychological terms. It serves to clarify such terms as are unfamiliar to the individual reader without burdening the text with a series of definitions with which he is familiar.

The first half of the book, written chiefly by Stevens, was designed to give the reader "the fundamentals of the science of sound—with a minimum of mathematics—and then to tell him what he hears when a sound reaches his ears, and what are the systematic relations between stimulus and sensation." The second half, written primarily by Davis, deals with the mechanism and electrical accompaniments of hearing and includes the functional anatomy and physiology of the ear.

Chapter 1 is a well written, but elementary, summary of the fundamentals of sound. The second chapter presents the standard material on sensitivity of the ear, with discerning comments, plus new work by Békésy on low tones and Stevens on electrical stimulation. Then follow in chapters three, four and five discussions of pitch, loudness, and other attributes of tones. A critical evaluation is made of the experimental work carried on in this country by the authors and other investigators, and by research workers abroad. All of the results are integrated in an excellent summary of the present state of knowledge concerning tonal attributes. The remainder of the first half considers auditory localization; aural harmonics and combination tones; auditory masking, fatigue and persistence; and modulation, vibrato and beats. Many valuable contributions to these subjects have been made by the authors and their collaborators at Harvard. Some of them are presented here for the first time.

The second half of the book is introduced by a complete

discussion of the mechanics of the ear (Chap. 10), followed by a discussion of deafness and bone conduction (Chap. 11). Chapter 12 gives a summary of the principles of neurophysiology. The next six chapters present a survey of the electrical accompaniments of hearing, starting with the now famous Wever-Bray phenomenon and discussing the electrical effects in different portions of the auditory mechanism, as follows: Chap. 13, the microphonic action of the cochlea; Chap. 14, considerations as to the nature and origin of aural microphonics; Chap. 15, the localization of the frequency-reception on the basilar membrane; Chap. 16, auditory nerve impulses, in general; Chap. 17, nerve impulses in response to tonal stimulation; and Chap. 18, nerve impulses in the higher auditory pathways. As in the first part, a masterly survey and critique of recent and contemporary work is made, giving large, but not undue, place to the authors' own contributions to the field.

In some instances the authors have felt it "appropriate to relate psychological with physiological functions." However, they feel that "an attempt, at present, to account for all psychological discriminations in terms of physiological processes is obviously premature, and explanations in this realm of psychophysiology must be cast in speculative form." They add that "Although speculation may be hazardous for the good repute of the speculator, it fulfills an important purpose when it serves to give perspective to a field of inquiry, or when it stimulates research designed to replace speculation by factual demonstrations." It is in this spirit that the authors have ventured "occasional suggestions as to possible psychophysiological correlations." Their suggestions are always well-considered and seem plausible. If future research proves them wrong, the authors will be the first to acknowledge it and to welcome the new facts. In this book they frankly admit in the light of present knowledge the fallacy of certain suggestions made in their earlier papers.

They have "surprisingly little to say about theories of hearing" yet their "inventory of the facts of audition" and their interpretations of them lead them finally to believe in "a place-theory that does not employ the principle of simple resonance" though they "did not begin with this type of theory in mind."

This book undoubtedly will inspire other investigators to delve further into the problems of audition and to apply the fundamental principles (more clearly understood because of the authors' efforts) to their specific fields of endeavor and to use them as guides in "future explorations of this vast and important field of research—a field in which our excursions have only just begun, but which holds rich rewards for man's irrepressible curiosity to know why."

NORMAN A. WATSON
University of California at Los Angeles

Current Publications on Acoustics

F. A. FIRESTONE

147 East Physics Building, University of Michigan, Ann Arbor, Michigan

Reviews of Contemporary Papers

MEMBERS who have occasion to translate important papers from foreign journals will be doing a real service to those members who do not read the foreign languages easily, if they will prepare a review of such articles for publication in this section.

These reviews should be of the nature of a lengthy abstract (500 to 1500 words) rather than a critique or appraisal, and should attempt to set forth in this limited space as much of the original author's contribution as possible. One or two figures may be included if desired.

To avoid duplication of effort it is desirable to arrange with the office of this department before writing a review.

Sound Absorption in Gases at Frequencies between 20 and 100 kc/sec. N. SCHMIDTMÜLLER, Akustische Zeits. **3**, 115–129 (1938). The classical theory of absorption, taking into account only friction and heat conduction, leads to absorption values which may be very much below those actually measured. Kneser[1] has shown that it is the oscillation within the molecule which produces the additional dissipation of energy. A great number of investigations have been carried out on this subject, as it is interesting not only from a practical but also from a theoretical point of view, promising some valuable information about the oscillations of the atoms in the molecule. The present measurements have been performed to close a gap in these investigations, extending from 20 to 100 kc/sec.

The measuring method most adequate for this purpose seemed to be the determination of the decrease of intensity in the free wave propagation. As source of sound a magnetostriction rod was used, with a fundamental of 5 kc/sec., vibrating in one of its harmonics, the receiver was a crystal microphone. Both were placed into a vacuum-tight boiler which was lined with hair felt to damp out the reflections from the walls. The diameter of the boiler was large enough (60 cm diam.) so that along its axis the free wave propagation was undisturbed by the diffraction of the wave in the proximity of the absorbent. The gaseous content of the boiler could be removed by evacuating it; then it would be filled with the gas to be investigated. The microphone was coupled with a recording arrangement, the whole working automatically. The gases used were of technical purity. There were no additional impurities to be expected because of an occlusion of the gases in the hair felt, as was shown by evacuating rapidly a vessel filled with such a material; gases liberated from the felt would have produced a gradual exponential increase of the pressure and such an increase could not be observed. However the hygroscopicity of the hair felt was disagreeable, as it did not allow the performance of measurements at a predetermined degree of moisture. Only afterwards the humidity could be determined by freezing out and weighing the water content of a certain volume of gas.

The magnetostriction rod was placed into a baffle which was large in comparison to the wave-length (vibrating disk in an infinite plane). Then it can be shown that the propagation along the axis of the rod at a sufficiently large distance from it obeys the laws of a spherical wave. Therefore, if m is the absorption constant, the following relation holds

$$Pr = P_0 r_0 e^{(-m/2)(r-r_0)},$$

P being the pressure and r the distance from the sound source. Taking the logarithm we get

$$\log_{10} Pr = -\frac{m}{2}(r-r_0)\log_{10} e + \log_{10} P_0 r_0$$

and this is a straight line relation, the slope being the absorption constant.

The first set of measurements was performed in nitrogen. The measured points were found to lie above the values calculated by the classical formula. The difference can be attributed to molecular absorption, but it could also be due to the proportionality factor in the classical formulas being too small by about 15 percent. Which of the two explanations is the correct one could only be decided if the measurements were repeated at much higher frequencies, for at high frequencies the molecular absorption reaches a constant value, which is small in comparison with the classical part in the absorption, which increases as the square of the frequency.

Figure 1 shows the measurements in oxygen, for the two values of humidity (*a*) 6,1 g/m³ (*b*) 17,2 g/m³. The dashed curve is calculated according to the classical formula. The

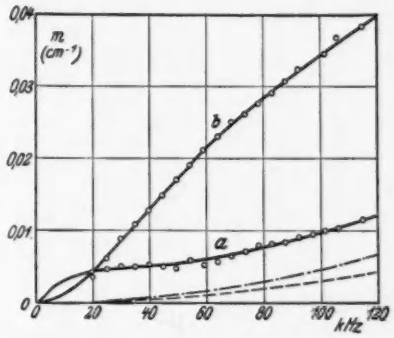


FIG. 1. Absorption of sound in oxygen, for the humidities (*a*) 6.1 (*b*) 17.2 g/m³.

difference between the two should be the molecular absorption which according to the theory is constant, if the frequency is large enough. The upper portion of curve (*a*) falls into this frequency range and should therefore be parallel to the dashed curve, which is not the case. But this difference can be made constant, if the proportionality factor in the classical formula is increased by about 50 percent. (Dot-dash line.) (Most of the modern investigations seem to show that the proportionality factor in the classical absorption actually is too small, the reason for this discrepancy being as yet unknown.) There is almost perfect agreement between the theoretical molecular absorption and the observed values; when they are plotted to a logarithmic frequency scale they can be brought to coincidence. However it has to be mentioned that the logarithmic frequency scale for the theoretical values is undetermined by a constant, which is not yet accessible to calculation.

Another set of measurements were performed in propane. In addition to the classical absorption there seems to be again a molecular absorption which is not very great, in spite of the complexity of the molecule.

The molecular absorption of carbon dioxide is very large and also depends greatly on the humidity. Measurements have been performed for the humidities 2.1, 4.7, 8.6 g/m³. If the absorption per wave-length is plotted against the frequency, a bell-shaped curve should result, the frequency of the maximum absorption depending on the humidity. The measured points covered only a portion of this bell-shaped curve, but there was almost perfect agreement with the theoretical values. An extrapolation of the measurements with the aid of the theoretical curve showed that the frequency of the maximum increases linearly with the humidity.

E. J. SKUDRZYK

[1] H. O. Kneser, J. Acous. Soc. Am. **5**, 122 (1933).

Subharmonics. E. WAETZMANN, K. KURTZ, Ann. d. Physik **31**, 661–680 (1938). Acoustic recording and reproducing systems produce a certain amount of nonlinear distortion, introducing harmonics, subharmonics and combination tones. In spite of their importance, only little attention has been payed up to the present to subharmonics. Even their existence seems to have been widely unknown. It has not yet been ultimately decided, whether subjective subharmonics may be generated by the human ear. Helmholtz wrote on this subject, but his considerations do not seem to be very probable to day. It appears that if subjective subharmonics do exist, they would have to be attributed to the nonsymmetric loading of the ear drum by its appendages.

Probably the best known example of the generation of subharmonics is Meldeś experiment. A fine string is maintained in transverse vibration by connecting one of its extremities with the vibrating prong of a tuning fork, the direction of motion of the point of attachment being parallel to the length of the string. The effect of the motion is to render the tension of the string periodically variable. If the string is tuned to the suboctave of the tuning fork, it will be found to start a vigorous vibration, whose period is the double of that of the fork. A tuning fork which just hits a piece of cardboard also produces strong subharmonics.

It is known that the output of a carbon microphone contains a certain percentage of subharmonics. As investigations show, these are generated purely mechanically, and can be heard even if there is no voltage applied to the microphone. One has only to place a tuning fork in front of the membrane, and to listen to the sound radiated from it. When a piece of paper covered with glass powder is placed into an intense supersonic field, a strong hissing tone is heard; this is a similar effect of frequency division. The subharmonics can be observed only if the amplitude of the primary sound is above a certain value. However, if the amplitude is too large the sounds will become impure and noisy. During the first excitation of the microphone they often appear weak and impure, repeating the excitation a few times they are found to become much stronger and purer. The particular distribution of the granules thus plays an important part. The membrane alone cannot produce undertones; the granules on the one side of it play a decisive part.

Records taken for a particular carbon microphone at a frequency of 2150 c/sec. showed the suboctave $n/2$ and and the frequency $n/4$ with the amplitudes 4 and 0.2 times that of the fundamental. Similar measurements were taken for a telephone. The subharmonics were found particularly strong, when the membrane was almost touching the magnet. In a particular case the frequencies n (2315 c/sec.) $n/3$, $2n/3$, $4n/3$ and $5n/3$ were found in the output of the telephone, their amplitudes being in the ratio 1 : 1.9 : 2.3 : 0.8 : 0.5.

The one dimensional analog of the electromagnetic telephone is a string excited at its midpoint by an electromagnet. If the string is tuned to a subharmonic of the exciting frequency, and the magnet approached close to the string, the latter will perform vigorous vibrations in its fundamental mode. As one dimensional analog to the carbon microphone, a cardboard edge is approached to the string until it just touches it. In this case also strong subharmonics will be observed.

Though in general subharmonics are considered as a resonance phenomenon, they do not depend greatly on the frequency. In most cases the exciting frequency could be varied within large limit without interrupting the generation of subharmonics.

As rough theoretical basis consider the equation of a vibrating mass:

$$m\ddot{x}+k\dot{x}+sx=F\sin\omega t.$$

It can be shown,[1] that subharmonics are possible, if at least one of the constants m, s, k becomes a function of x, for instance

$$F=6(1+gx) \qquad \text{or} \qquad s=s_0(1+\alpha x).$$

In case of the carbon microphone, the stiffness becomes a function of x. For the electromagnetic telephone, as approximation of the second order, s and F are found to depend upon x in the above manner.

E. J. SKUDRZYK

[1] *Subharmonics in Forced Oscillations in Dissipative Systems.* P. O. Pedersen, Kopenhagen. 1933.

Simple Tests of Materials with Magnetostrictive Supersonic Apparatus. E. MEYER, G. BUCHMANN, Akustische Zeits. **3**, 132–136 (1938).

(a) The determination of cracks

In the course of some investigations with ferro-concrete bars two of them were found to have a fundamental of 33 and 48 c/sec. respectively. The two bars were exactly of the same type except that the one with the lower natural frequency had been loaded during some preliminary tests. The difference between the two natural frequencies could therefore be attributed only to internal faults of the bars.

To certify this, the bars were excited to longitudinal supersonic vibrations. Transmitter and receiver were of the magnetostrictive type, consisting of a number of parallel metal sheets (the core being similar to that of an ordinary transformer), carrying a low resistance coil. They were plastered on to the bars. In this way a solid and reproducible connection resulted. (The coupling could also be done with a layer of water, the thickness of which would have to be a multiple of $\lambda/2$, to get the best matching possible.) The magnetostrictive system was fed from an ordinary valve oscillator and could be excited to vibrations within the range 21–23½ kc/sec. In this range, the transmission of the two bars differed greatly, at an average, the amplitudes transmitted by the bar with the lower fundamental was 50 times smaller than those transmitted by the second bar. Moving about the receiver then led to the approximate location of the faulty place. This shows that tests of the type described can be efficiently performed in the lower supersonic range, with a very simple and solid apparatus.

(b) The setting of cement

A mixture of Portland cement with 26 percent of water was placed between transmitter and receiver, forming a layer of 4 cm thickness. The transmission during the setting was measured, showing that the reaction started after about 50 minutes and terminated after a time of the order of 300 minutes. These measurements included the effect of the growing on of the cement on the receiver and transmitter surfaces. In a second test, a liquid coupling was used therefore. The cement mixture was placed into turpentine, where it was insoluble, whilst the transmitter and receiver, owing to the solubility of the rubber insulation, were immersed into water. The two liquids were separated by a thin metal sheet. The results obtained in 4 similar cases are shown in Fig. 1. The reaction is seen to start after 200 min. and is terminated after about 800 min.

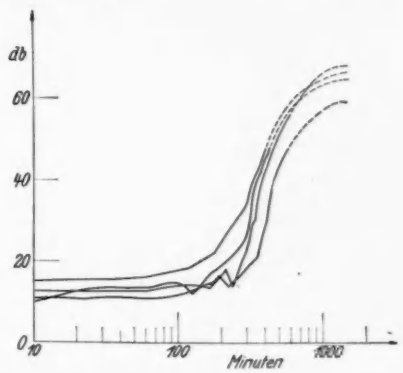


FIG. 1. Transmission of sound by a mixture of setting cement.

(c) The influence of absorbers on the propagation of sound

During the propagation of sound along an absorbing surface the wave front is bent towards the absorbent and the intensity decreases at a greater rate than in the undisturbed wave. Because of the convenient dimensions, a frequency of 20 kc/sec. was used to investigate this phenomenon. A magnetostriction rod served as sound source, whilst the receiver was a Rayleigh disk suspended on a platin fiber. The disk tends to take up a position perpendicular to the particle velocity. The torsion head of the suspension was therefore turned such that the disk remained stationary when the sender was switched on and off. The direction of propagation could then be read off directly from the position of the torsion head. During the measurement, the distance from sound source to Rayleigh disk was kept constant, whilst the absorbent was moved towards the disk. The largest deviations from the direction of propagation of the undisturbed wave thus found varied between 4 and 9 degrees, according to the absorbent, whilst the corresponding decrease in intensity was about 4 db. This diffraction effect could only be observed in the immediate proximity of the absorbent. They were limited in all cases to the space of one wave-length.

E. J. SKUDRZYK

A New Method for Tuning Pianos. JEAN DE BREMAEKER, Musique et Instruments, **28**, 179–183, 209–211, 243 (1937).—In a comparison of just and tempered intervals it is pointed out that three successive just major thirds (5/4) do not give a true octave (2/1). Three tempered major thirds ($2^{4/12}/1$) do give a true octave (2/1). Two major thirds, each comprising two whole tones, form an augmented or "whole tone" fifth.

A scale built up by selection from a series of harmonics is compared with the equally-tempered scale, attention being directed to that note derived from the thirteenth harmonic and lying between the tempered augmented fifth and the tempered major sixth.

In an unusual approach to the recognition of intervals by the ear, it is stated that the determining factor is the difference in vibration rates of the two notes forming the interval, and its relationship to the vibration rate of the tonic or lower tone. Above the fifth, the characterizing difference is to be computed with respect to the octave of the tonic; above the twelfth, with respect to the double octave. Thus the fifth gives the largest difference, this being the reason for its "fullness" to the ear.

The interval of the tempered tenth, which gives prominent beats, may be used as the basis of a method of laying the temperament. Three successive tenths, properly mistuned from the just interval and carried back by octaves, give the original tonic, thus furnishing a control which can be applied sooner in the process than that of the classical method based on twelve indirect fifths. Since the tenth is the major third on the octave of the tonic, and the major

FIG. 1.

third is composed of two whole tones, the method of tuning is given the name, "whole tone tuning."

Figure 1 shows the tuning procedure in staff notation. In each space, the half note is the one which has already been obtained and the quarter note is the one which is to be tuned by comparison with it. Beneath the space is given the number of beats per second* which are heard when the interval is properly tuned in conformity with equal temperament. The process is divided into four parts, each of which gives three notes. These three notes are indicated at the right, and it will be observed that in each case they can be arranged to form an augmented triad (two major thirds). In the second part and in the third part it is necessary to tune a tempered fifth as a base for the other intervals. While the tempered fifth is difficult to tune correctly, the fact that only two need be taken as bases gives less chance for error than the classical method using only fifths and fourths.

The method given requires that:

1. The octaves be tuned free of beats,
2. The major tenths be tuned sharp (to give the beat rates shown),
3. The fifths be tuned slightly flat.

True octaves are to be tuned throughout. It has been objected that this is of value only for instruments such as organ and harmonium, since pianos and harpsichords are generally not tuned to true octaves at the high and low ends. The author is strongly opposed to this *barbarous practice*, whereby brilliant music is obtained at the expense of pathetic.

ROBERT W. YOUNG
O. HUGO SCHUCK

* Reviewers' note: The beat rates given in the figure have been recalculated on the basis of the American standard pitch of $A = 440$ cycles per second. Beat rates for the fifths have been included, the author only indicating that they should be of the order of one or two per second. It would seem that the rather high beat rates of some of the tenths might profitably be reduced by using notes an octave lower.

To compute beat rates, the author obtains the difference between the vibration rate of the upper note of the interval, when calculated for just intonation, and that for equally tempered intonation. The author, "without understanding the mechanism of the law," finds it necessary sometimes to use the octave of the note determining the interval. His calculation of the vibration rate difference between the appropriate harmonics of the notes concerned gives twice the observed beat rate, but this may be explained as the result of the author's use of the number of single vibrations. Since the ear counts only the reinforcements, it will hear a number of pulses per second equal to the difference between the numbers of whole vibrations or cycles per second. Expressing the frequencies in cycles per second, calculation of the differences between appropriate harmonics of the notes sounded gives the observed beat rate directly.

In reading the original article, it should be remembered that the author uses "harmonique" as overtone, i.e., the "1° harmonique" is the first overtone, or second harmonic, since its frequency is twice that of the fundamental.

References to Contemporary Papers on Acoustics

MOST of the titles of papers appearing in other languages than English have been translated; the clue as to the language in which the article is written can usually be found by noting the name of the journal in which it appears. Abstracts in English of many foreign papers have appeared or soon will appear in Science Abstracts, Section A. Where references are made to Science Abstracts, the reference is to the volume number and abstract number. The abbreviations of the names of journals are those used in Science Abstracts and can be found in any annual index to those abstracts.

The numbers appearing at the head of each entry are the numbers which identify the subject headings in the Cumulative Index of this journal which is to appear soon.

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